quest for an abstraction called the "true" intensity while depreciating the actual observation as "apparent" intensity!

Intensities, particularly in regions where the earthquake was slight, must usually be rated from written reports of local observers. In such cases much depends on the reporter. If he inquires among his acquaintances, he may give a fair value for the intensity. If he gives only his personal experience, his report may not be so valuable. The United States Coast and Geodetic Survey issues a post-card questionnaire which is circulated widely after earthquakes. On this the observer may check items regarding the intensity in his locality like those used in fixing the grade on the Modified Mercalli Scale. It is necessary that reporters be guided by such questionnaires. Many an individual who considers himself much interested in earthquakes and is willing to write a long letter reporting a shock actually devotes the whole report to his theory as to what caused it.

When the intensity in each locality has been rated, the values are usually plotted on a map. "Isoseismal" lines are then drawn so as to separate areas of one intensity from those of another (Figure 19). In general the resulting "contours" show a high at the center, with regions of lower intensity surrounding this area and more or less concentric with it. Some seismologists feel that unless the isoseismals are smooth curves concentric to a point, the whole process has been valueless. This feeling harks back to the days when locating the epicenter by this means was the sole approach to it. To the author the interesting feature of the isoseismal map is its irregularity. The irregularities point

SEISMOLOGY

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SEISMOLOGY

By

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Preface

In this volume I have attempted to cover in brief the field of earthquake seismology.

Applied seismology or seismic prospecting has been treated in another volume of this series. The general subject of earthquakes, the theory of earthquake waves, and the theory of the seismograph have been treated *in extensio* in other books. The effort here has therefore been directed toward brevity.

The first seven chapters, Part I, are on the subject of earthquakes and require little technical knowledge. Part II, entitled Seismography, presupposes readers with some preparation in mathematics and physics.

A few of the photographs reproduced in the illustrations are of unknown origin. They are a part of the file of photographs accumulated through many years at the Berkeley seismographic station. Where the source is known, acknowledgment is given.

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EARTHQUAKES

CHAPTER I

Elasticity and Plasticity

Before beginning a discussion of the phenomena observed in connection with earthquakes, it is necessary to agree on the meanings of terms which must be used in that discussion.

Elastic constants

When solid bodies such as rocks at ordinary temperatures are subjected to moderate applied forces, they are deformed; and when the applied forces are removed, the bodies return to the original forms. A body is said to possess *elasticity* or to be *elastic* according to its resistance to this type of deformation. The greater the resistance of a body to deformation, the more elastic it is said to be. This is not the popular idea of elasticity, for rubber and not steel comes to the mind of most people when a highly elastic body is mentioned.

The forces per unit area set up inside the body to resist deformation are called *stresses*. At any point in the deformed body the value of the stress depends upon the orientation of the plane upon which the area is measured. A complete description of the stress at a given time at a given point in a body requires the statement of each of three mutually perpendicular components of the stress (one normal and two tangential) across each of three mutually perpendicular planes through the point. It develops that there are equalities among the tangential components which reduce the number of quantities necessary to describe the stress from nine to six. The deformation of the body accompanying stress is called *strain*. Thus stress and strain occur together.

There are two general types of deformation of which all kinds of elastic deformation may be compounded. The first is a change in volume without any change in shape. If an isotropic body is placed in a fluid medium and the fluid is compressed, the body is subjected to *hydrostatic* pressure, that is, a pressure which is applied equally and normally to all its bounding surfaces (Pascal's Principle). The result is a change in volume but no change in shape. The strain set up in the body in such a case is called *dilatation.* There are two kinds of dilatation: *compression,* in which the volume is reduced; and *rarefaction,* in which it is increased. The latter is produced by reducing the hydrostatic pressure below its normal value.

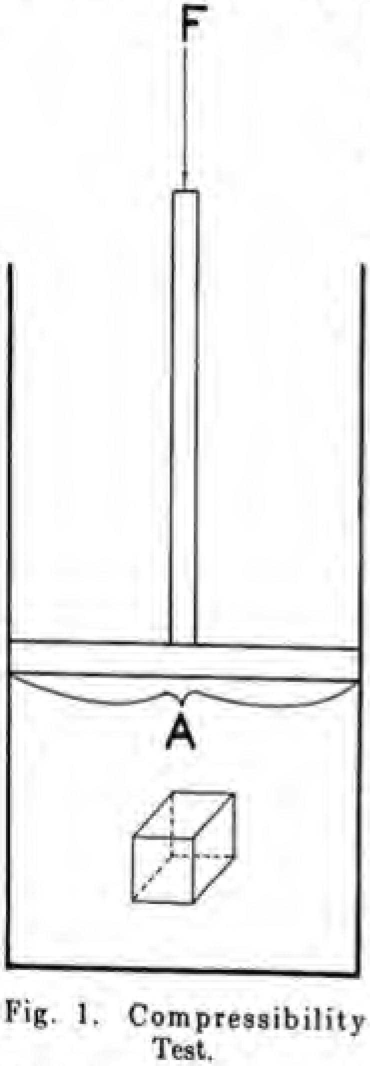

Fig. 1. Compressibility Test.

The second type of elastic deformation is a change of shape without a change in volume. A body with a square cross section may be strained by the application of a couple into a body whose cross section is a rhombus, the rhombus having the same area as the square. This type of strain is called *shear.* It is most commonly encountered in everyday life when bodies are "twisted." There are thus two kinds of elasticity, incompressibility and rigidity. The former is a measure of the body's resistance to change in volume, and the second is a measure of its resistance to change in shape.

Hooke's Law, named after the man who first made a statement of it, declares that, within limits, stress is proportional to strain. It is within the limits for which Hooke's Law holds that the classicial Theory of Elasticity

applies. This theory is discussed in a later chapter. Hooke's Law is used in defining the *elastic constants* of a material. These give a quantitative measure of elasticity. Referring to the conceptual experiment shown in Figure 1, the body whose constants are to be tested may be placed in a cylinder filled with fluid and the fluid compressed. By Pascal's Principle the pressure (force per unit area applied normally) applied by the piston head is equal to the pressure applied to all surfaces of the test piece. It is therefore a measure of the internal forces per unit area set

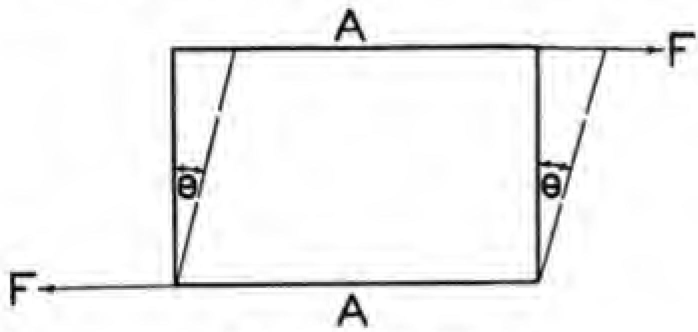

Fig. 2. Shear Test.

up in the piece. If F/A is the force per unit area exerted by the piston head, and if strain is measured by $\Delta V/V$, the change in volume per unit volume of the test piece, we can write Hooke's Law for this case as

$$\frac{F}{A} = \kappa \frac{\Delta V}{V}.$$

κ is called the *coefficient of incompressibility* or the *bulk modulus*. It is one of the elastic constants of the material and is a measure of its resistance to change in volume. To define another elastic constant, reference may be made to the conceptual experiment shown in Figure 2. A body with a rectangular cross section has been strained by a shearing couple into one of a cross section that is a rhomboid. The rhomboid and rectangle have the same altitude and therefore the same area. The stress is measured by F/A (where F now acts parallel to area A), and the strain

by the angle θ (θ is an approximation for $\tan\theta$ where θ is small). Hooke's Law then gives

$$\frac{F}{A} = \mu\theta,$$

where μ is called the *coefficient of rigidity* or *shear modulus*.

Another elastic constant describes the reaction of the material to linear extensions or contractions. Referring to Figure 3, a test bar may be stretched (or compressed) by applying a force in one direction only. If force F is applied over area A, an increase (or decrease) of length Δl is observed and a change in diameter Δd. F/A is a measure of the stress, and $\Delta l/l$ of the strain. Then, by Hooke's Law,

$$\frac{F}{A} = E\frac{\Delta l}{l},$$

and E is called *Young's Modulus*. The ratio of the lateral contraction to the longitudinal extension $\Delta d/d \div \Delta l/l$ is called *Poisson's Ratio*.

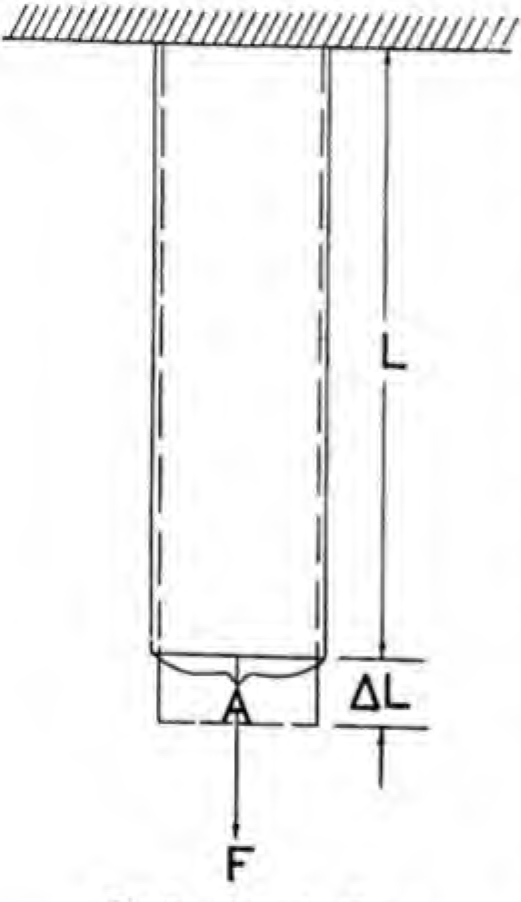

Fig. 3. Extension Test.

The above elastic constants are used in describing the reaction of rocks to stresses and are of interest in seismology because an earthquake is transmitted by elastic waves and the properties of the wave in any medium depend partially on these constants.

Hooke's Law, as stated above, takes account of the fact that there is a limiting stress above which the linear relation between stress and strain does not hold. This

limiting stress is called the *limit of linear elasticity*, or sometimes merely the *elastic limit*. (The elastic limit is perhaps better defined as that limiting stress which a body can bear without being permanently deformed.) Very careful observations show some small departures from Hooke's Law even below the limit of linear elasticity, but such are very small, second-order quantities. A *brittle* substance breaks when the applied stress reaches the elastic limit. Most rocks at the surface of the earth are brittle, and therefore one sometimes meets the expression: "the rocks reach their elastic limit and break." At depths in the earth where temperatures are higher, rocks are probably not brittle. A material which is not brittle may be strained above the elastic limit without rupture. Above this limit an added increment of stress produces more strain than an equal increment at lesser stress. The material may rupture above the limit of linear elasticity. The stress at which it ruptures is called the *breaking stress*.

Plastic flow

If a substance is strained above its limit of linear elasticity and the applied force is removed, it may not return to its original form. It is then said to have experienced *set*. The limiting stress above which the material experiences set is called the *set point*. If the set does not decrease with time after the load is removed, it is called *permanent set*.

If a body is subjected to a shearing stress sufficiently great, under some conditions it may not rupture but may continue to accumulate set at a uniform rate. The stress which accomplishes this is called the *strength* of the material, and the phenomenon, *plastic flow*. It is frequently presumed that rocks at moderate depths in the earth deform in this fashion. An elastic body stressed beyond its strength may experience plastic flow, but the removal of the stresses will be followed by some elastic recovery, although the new position of rest of its parts relative to

each other will be different from the old by an amount equal to the set. If a rigid body has very little strength but a very high viscosity, the application of high frequency oscillatory stresses in excess of the strength will be met by rigidity, and the body will vibrate without having time to accumulate set. When stresses below the elastic limit are applied for long periods of time, there may be plastic flow or *creep*[1]. The distinction between elasticity and plasticity becomes somewhat confused when very long time intervals are considered.

In discussing the conditions in the interior of the earth a distinction between rigidity and strength must be kept in mind. A body may have insufficient strength to stand even its own weight and thus be unable to retain its shape for a long period of time. Yet it may be quite rigid to high frequency oscillations even while experiencing slow plastic flow. Jeffreys[2] cites the example of shoemaker's wax, which is so rigid that a tuning fork may be made of it. However, if left to stand, the fork will lose its form and flow down into a flat mass.

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CHAPTER II
Earthquake Vibrations

Definitions

It has long been recognized that earthquakes are vibrations in the elastic rocks of the earth. An earthquake may be defined as an elastic vibration of the earth, but further qualifications must be made. Sensitive seismographs show that the earth's crust is in a continuous state of vibration. These continuous vibrations are called *microseisms* or *earth unrest* and are caused by the works of man, such as machinery; by atmospheric action, such as wind and perhaps variations in atmospheric pressure; by water action, such as waves on the coast, and so forth. All these are more or less continuous phenomena. The earthquake is a transient thing. The greatest of shocks is felt for no more than a few minutes, although sensitive seismographs at a distance may record for hours earthquake waves traveling by varied paths and arriving at different times. It is by the sharp division of vibrations into groups of waves that an earthquake stands distinct from microseisms on a seismogram. Earthquakes may be subdivided into two classes: natural and artificial. The latter may be caused by explosions or falling weights. The former are the subject of discussion here. The source of the waves which are the earthquake is called the *focus* of the shock. The nature of the focus will be discussed in Chapter III. The place on the surface of the earth directly above the focus is called the *epicenter*.

Simple harmonic motion

Although earthquake waves are very irregular, it is useful in a discussion of them to compare them with some simple

type of vibration which can be handled with ease mathematically. *Simple harmonic motion* is useful in this respect. Simple harmonic motion is a kind of *periodic motion,* that is, it repeats itself at constant intervals of time. Simple harmonic motion is periodic motion in a straight line such that the acceleration is always directed toward the center of path and is proportional to the distance from that center.

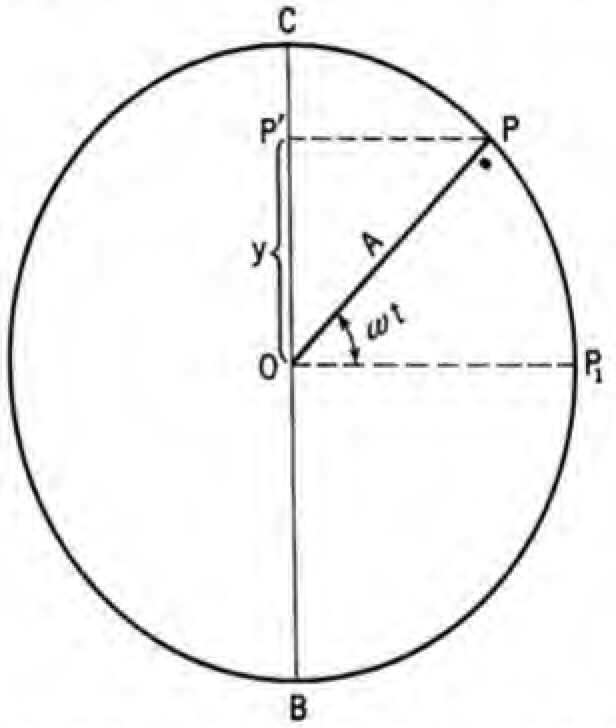

Fig. 4. Reference Circle for Simple Harmonic Motion.

The motion is perhaps easier to visualize when defined as follows: If a particle moves with uniform speed around the circumference of a circle, its projection on any diameter of the circle moves with simple harmonic motion. Let P in Figure 4 be a point moving with uniform angular speed ω around the circumference of the circle of radius A in a counterclockwise direction from a starting point P_1. Then the position of P', the projection of P on diameter BC, is described by

$$y = A \sin \omega t,$$

where t is the time measured from a zero when P was at P_1. Point P' executes simple harmonic motion; y is its *displace-*

ment from its median position O; and ω is the angular velocity of the reference particle P and is measured in radians per second. Let T represent the time taken by P in a complete revolution, that is, the time taken by P' in traveling the diameter twice. A complete revolution involves an angular change of 2π radians. Therefore,

$$T = \frac{2\pi}{\omega}.$$

T is called the *period* of the motion, A its *amplitude*, and ωt its *phase*. The motion of P' is a fair approximation to that of an earth particle in some simple types of earthquake waves. It frequently has been used in the past as a rough approximation of more complex earth waves, although modern seismology tends more and more to abandon such rough approximations. The velocity of P' may be obtained by differentiation:

$$\frac{dy}{dt} = A\omega \cos \omega t = A\omega \sin \left(\omega t + \frac{\pi}{2}\right).$$

Thus the velocity of P' is zero when the displacement is a maximum, and is a maximum when the displacement is a minimum. The velocity leads the displacement by 90°. This velocity may be called the *molar velocity*, that is, the velocity of a particle to distinguish it from the speed of propagation of the wave.

The acceleration of P is

$$\frac{d^2y}{dt^2} = -A\omega^2 \sin \omega t.$$

It has its maximum positive value when the displacement has its largest negative value, and its largest negative value when the displacement has its maximum positive value. The acceleration and displacement are therefore 180° out of phase. In Figure 5 the displacement, velocity and acceleration of P' are plotted as a function of time for the case of $\omega = 1.5$. Any one of these curves may be taken to represent a seismogram of an earthquake in which the ground

executed simple harmonic motion. Depending on the relation of the period of the earth wave to the period of the seismometer pendulum, the response of the instrument and thence the seismogram displacements may be proportional to the earth displacement or the earth velocity or the earth acceleration.

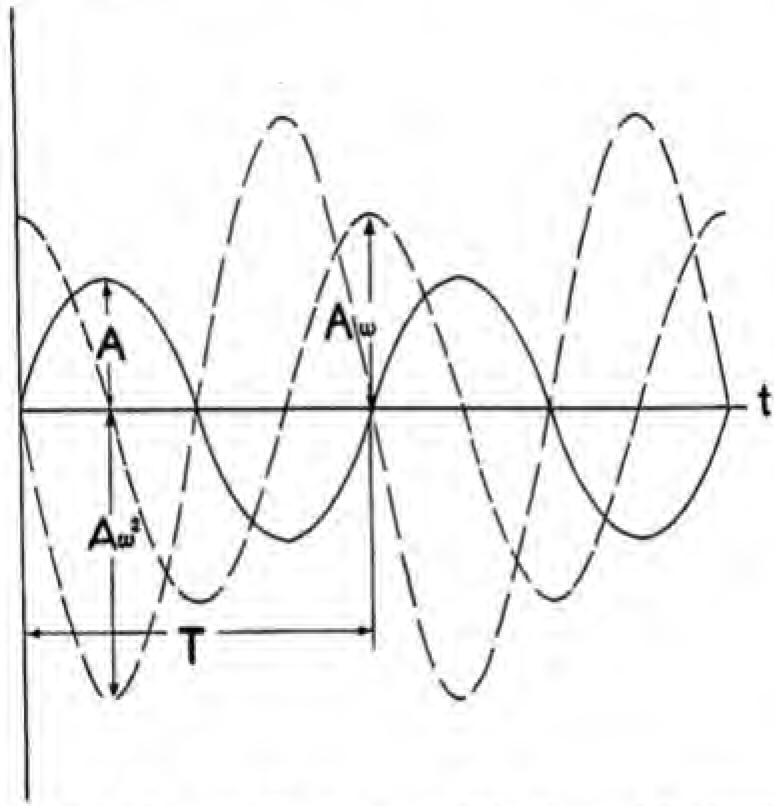

Fig. 5. Simple Harmonic Displacements, Velocities, and Acceleration as a Function of Time.

On Figure 5 the time, T, gives the period of the motion, while A represents the *amplitude* of the displacement.

Although earthquake waves are frequently irregular and not strictly periodic, they may often be approximated (sometimes only roughly) by simple harmonic waves and the "periods" and "amplitudes" measured.

Earthquake waves

A conspicuous character of a seismogram is the division of the vibrations into definite groups which begin quite sharply. Figure 6 shows a seismogram recorded at Berkeley

of the earthquake of Nov. 25, 1941. The record may be divided into two parts, as was done by early seismologists. The first groups of smaller amplitude are called the *preliminary waves*, and the later groups of larger amplitude the

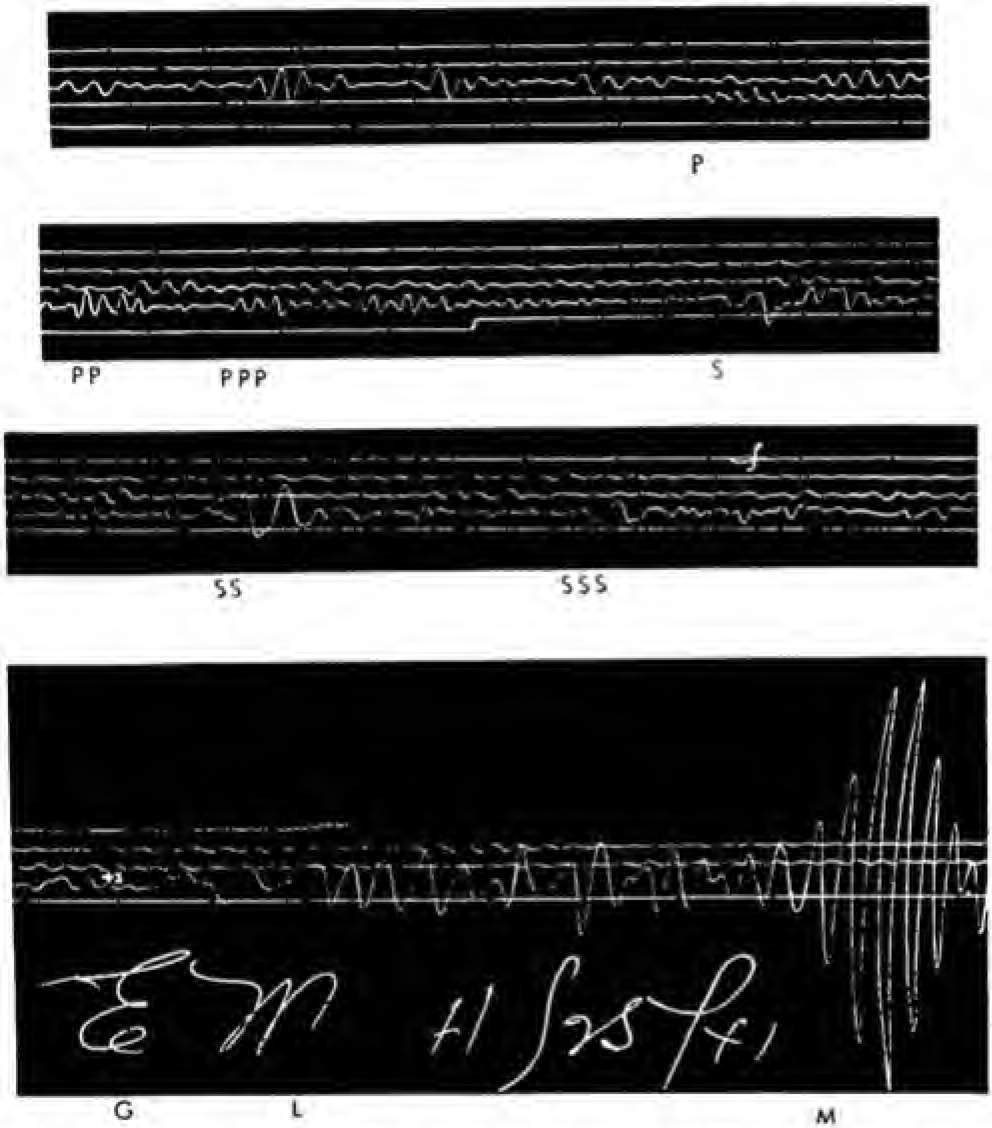

Fig. 6. Berkeley Seismogram (Bosch-Omori, East-West Component) of November 25, 1941, Epicentral Distance about 6,000 Miles.

principal portion. On some modern short period seismographs these later waves of large earth amplitude are neglected and, from the appearance of the seismogram alone, are not the principal portion. But it is these later

waves which carry the large earth amplitudes in most earthquakes. On the records of a large distant earthquake, motion persists for a long time after the principal portion. The waves are usually sinusoidal and frequently die down and build up in the nature of beats. These later waves are called the *coda*.

Preliminary waves

Long before the analysis of earthquake records was attempted, mathematical physicists had developed the theory of wave motion in elastic media, although their great purpose was to explain the propagation of light in the "ether." Theory shows that a sudden disturbance of the elastic equilibrium at a point in an elastic medium will result in the propagation from that point of two kinds of elastic waves. These two kinds of waves will propagate with different speeds. The first type consists of waves which are dilatational—during their passage the transmitting medium alternately experiencing compression and rarefaction, the volume changing, but not the shape. That is the physical description of the wave. Geometrically it is called a *longitudinal* wave because the particles of the material vibrate back and forth along the path. The *speed of propagation* of this wave is

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\mu}{\rho}},$$

where κ and μ are the coefficients of incompressibility and rigidity respectively and ρ is the *density* or mass per unit volume of the transmitting medium. The speed of propagation must be sharply differentiated from the *molar velocity* of the particles of the medium as the wave passes. The molar velocity depends merely on the amplitude and frequency of the wave and was defined above for the case of simple harmonic motion. The speed of propagation, that is, the speed with which the elastic disturbance is transferred through the medium, is independent of the amplitude

and period of the wave motion and therefore of the speed with which individual particles vibrate about their positions of rest.

The second type of elastic wave is the shear wave. As it passes, the medium changes shape but not volume. Geometrically it is a *transverse* wave (the particles vibrate at right angles to the path of the wave). The speed of propagation of these waves is

$$V = \sqrt{\frac{\mu}{\rho}}.$$

Thus the longitudinal waves travel faster than the transverse waves. Now the preliminary waves on earthquake records consist of two main groups. The earlier ones are called P waves, P standing for "primus." These are longitudinal waves. The later preliminary waves are called S for "secundus" and are transverse waves. The nature of the vibrations in these waves is established by the analysis of seismograms.

Travel time curves may be drawn for P and S waves by plotting the epicentral distances of the stations against the time of arrival of the waves at those stations. Such curves are plotted in Figure 49. The epicentral distance is measured around the surface of the earth. The fact that these curves are concave downward indicates that the waves do not travel along the surface of the earth. Elastic theory indicates that the speeds of longitudinal and transverse waves depend only on the elastic constants and densities of the rocks traversed and not at all on the time elapsed since the wave started. If the P and S waves traveled in the surface, the travel time curves would be straight lines, that is, the time of travel of the waves would be proportional to epicentral distance. True there would be some scattering of observational points about the straight lines because the surficial rocks vary somewhat in different localities, but with epicenters in different parts of the earth and waves transmitted by different paths a composite

curve for many shocks would give an average straight line curve.

Now for *P* and *S* curves built up from many shocks there is surprisingly little scattering. And the apparent increase in speed of travel as epicentral distance increases is very large. If these waves do not travel along the earth's surface, the next suspected path would be along chords of the earth. But if the travel times of *P* and *S* are plotted

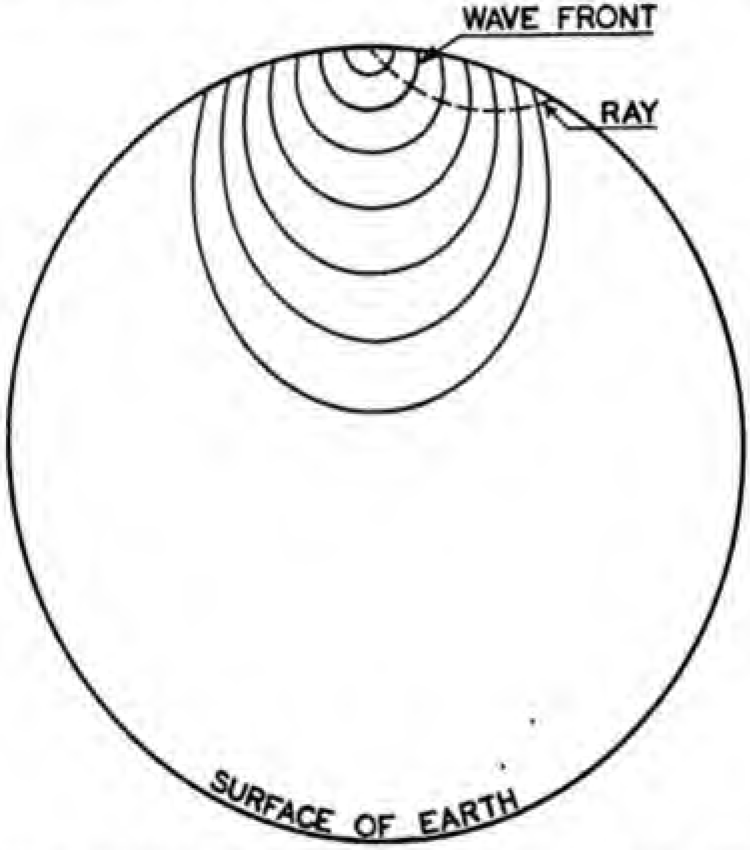

Fig. 7. Wave Fronts and Ray in an Earth in which Speed Increases with Depth.

against the chord distances from epicenter to stations, the resulting curves are still concave downward although the curvature is not so great as it was before. The suggestion then is that the deeper the chord the greater the speed along it. This means an increase of speed with depth, which would immediately result in curved paths, not straight ones such as chords. Figure 7 shows diagrammatically the effect on wave fronts and rays of an increase of speed with

depth. The former are warped from spherical form. Consequently the wave energy would return to the surface of the earth by curved paths.

Thus the form of the travel time curves of P and S has led to an important conclusion regarding the interior of the earth, namely: the speed of elastic waves increases with depth. But we know from astronomical data that the density of the earth as a whole is about 5.6. The rocks of the earth's crust have values only a little greater than half this. (See Table 1.)

TABLE 1

VALUES OF ELASTIC CONSTANTS AND DENSITIES FOR ROCKS FROM IDE[1]

Rock	$\kappa \times 10^{-11}$	$\mu \times 10^{-11}$	ρ
Granite (Quincy)	4.05	1.80	2.63
Olivine diabase (Vinal Haven)	6.85	4.19	2.97 ca.
Norite (Sudbury)	6.06	3.89	2.86 ca.
Dolomite (Pennsylvania)	8.40	3.62	2.83

Therefore the density of rocks in the earth's interior must be greater than that of rocks at the surface. Now density occurs in the denominator of the expressions for the speeds of P and S waves. Therefore the elastic constants κ and μ must increase more rapidly with depth than does ρ, the density.

It is frequently stated that the speed of elastic waves increases with depth in the earth *because* the density increases. This is a misstatement. Rather it should be said: The speed of elastic waves increases with depth in the earth *despite* the increase in density.

Laboratory experiments to determine the values of κ and μ for various rocks on the earth's surface have shown that rocks of higher density have correspondingly much higher incompressibility and rigidity, so that the quotients

$$\sqrt{\frac{\kappa + \frac{4}{3}\mu}{\rho}} \quad \text{and} \quad \sqrt{\frac{\mu}{\rho}}$$

are higher for rocks of larger density.

The preceding discussion of travel time curves of P and S holds for epicentral distances out to about 103° (where

these distances are measured by the angle subtended at the center of the earth). P and S waves emerging at this distance have paths which have just grazed the earth's central core. The increase of speed with depth then holds down to the core boundary at about 2,900 kilometers.

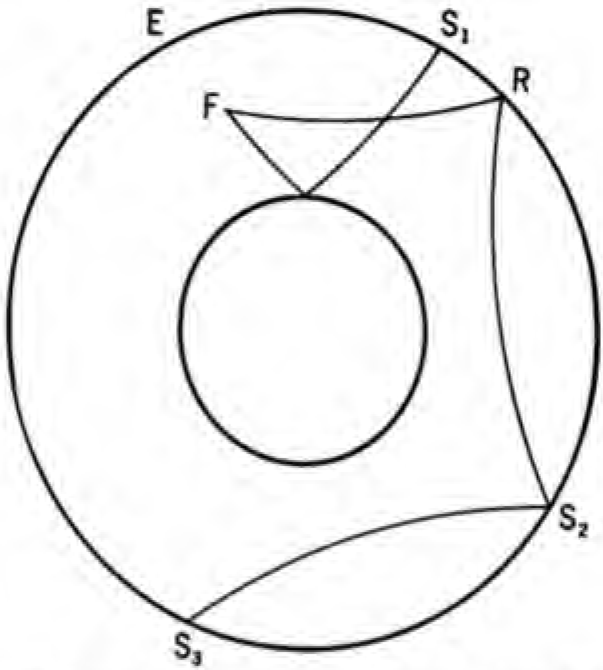

Fig. 8. Paths of Waves Reflected at the Earth's Surface and at the Core Boundary.

A glance at Figure 49 indicates a quick method of locating the epicenter of an earthquake if the records from three or more stations are available. The interval in time between the arrival of P and S at any station determines the distance from that station. The distance from the station determines a circle on the earth's surface upon which the epicenter lies. The intersection of three such circles determines the epicenter.

Between P and S other wave groups are conspicuous at certain epicentral distances. These are found to be reflections of P both at the surface of the earth and at the core boundary. Figure 8 indicates the paths of some of these reflections. From the focus F there goes out a P wave which is reflected at the core and returns to the surface at S_1. There also goes out from F a P wave which is reflected

at the earth's surface at R and again returns to the surface at S_2. It is reflected again and again emerges at the surface at S_3.

The S waves experience reflections just as do the P waves. Moreover, when either P or S are incident on a reflecting surface, generally there are generated reflected waves of both types. Refraction occurs at the core boundary with two possible refracted waves, P and S. It has not yet been established that any S waves exist in the core, and there is therefore strong evidence that the rigidity of the core is zero. P waves inside the core may be reflected at the inner side of the core boundary. Thus the preliminary waves may be transformed in type, reflected, and refracted in many ways, with the result that very many wave groups are present on the seismogram.

Surface waves

The principal portion on the seismogram consists of two or three groups of waves. If the arrival times of one of these groups are plotted on a travel time graph, it is found that they describe a straight line travel time curve, showing that they are surface waves. There is a great deal of scattering of points about the line, many points lying a minute or more from the line. There are two reasons for this scattering: first, the rocks on the earth's crust vary somewhat over the earth's surface, and thus the speeds of propagation vary somewhat with the path; second, the surface waves exhibit the phenomenon of dispersion—variation of speed with the period of the wave.

The fastest group of surface waves records only on the seismographs which write horizontal motion. These waves are transverse and have no vertical component. Their periods are long, varying from, say, 20 seconds to several minutes. They are called *Love waves*, after Love, who first developed the mathematical theory of their propagation[2]. Love showed that if the earth's surface consists of a layer of lower-speed rock overlying a medium of higher speed,

then transverse surface waves with no vertical component should exist. The speed of very short Love waves should be the speed of S waves in the surface layer, and the speed of very long Love waves should be the speed of S waves in the lower medium. Meissner[3] later showed that a mere increase of speed with depth, without layering, also accommodates a surface wave of the Love type. On some seismograms there is no sudden change in appearance of the waves after the beginning of the Love waves, the amplitudes gradually increasing to a maximum. On the other hand (see Figure 6), there is sometimes an abrupt change from the Love waves (referred to as G) to the maximum group (called M). And again some records show between the long G waves and the large M waves an irregular group of waves with some vertical component. These have been called L waves where their travel times have been listed.

Lord Rayleigh[4] developed the earliest theory for surface waves. He considered the free surface of an isotropic medium and showed that such a surface would support surface waves of such a character that during their passage the particles at the surface would vibrate in ellipses in a vertical plane containing the path. The major axis would be vertical and the motion retrograde. These waves are more or less approximated by the maximum group of waves on the seismogram and by the coda.

The surface waves are of great prominence on most seismograms of normal earthquakes. Only on very short period instruments (one second or less) are they sometimes neglected on account of their relatively low acceleration. Very deep focus shocks, however, naturally fail to set up large surface waves. The first indication that a record is one of a deep focus shock is the weakness of the principal portion.

Complicated character of earthquake motion

Later in this book there is a more detailed discussion of the various types of waves, their mathematical theory, and

conclusions to be drawn from their paths as to the nature of the interior of the earth. It suffices now to emphasize the very complicated nature of earthquake motion due to the varied types of vibrations and frequencies in the different groups. Particularly near the source of the shock, where many groups of waves arrive almost at the same time, it can be expected that the ground motion will be very complicated. Records of low magnification seismographs and observations of effects on structures confirm this expectation. No very simple theory of ground vibration may be expected to explain the damage to structures during an earthquake.

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CHAPTER III

Immediate Cause of Earthquakes

Faulting

A few great earthquakes of history have been accompanied by fractures of the earth's rocky crust along long lines or narrow linear zones. In many of these cases there was considerable displacement of the crust on either side of the breaks. The depths to which these breaks penetrated are not known. Such ruptures of the earth's crust are known as *faults*.

Field geologists observe in great numbers faults which have not exhibited surface displacements in the memory of man but which have obviously experienced them in the past, as is evidenced by the displacements of natural physiographic features as they cross the faults. Figure 9 shows a stream channel displaced where it crosses the San Andreas Fault in the Carrizo Plains region of California. Note the linearity of the fault, so conspicuous from the air. The existence of old faults is evidenced not only by physiographic features but also by the rocks themselves. The formations on the two sides of a fault are frequently of quite different age and rock type, the fault displacements having brought into juxtaposition strata originally not so situated. Figure 10 shows an exposure of the San Andreas Fault beside a road between Watsonville and Chittenden. To the left the rock is granodiorite, while to the right are Tertiary shales.

The breaking of the earth's crust along faults is assigned as the immediate cause of most earthquakes because (a) a number of large earthquakes have been accompanied by visible surface faulting, and (b) there is a tendency for epicenters located by use of seismographs and the regions

of maximum intensity of earthquakes to lie along faults recognized geologically even though at the times of the earthquakes these faults showed no surface ruptures. Under case (b) it is assumed that the faults have broken at depth but that the tear has not extended to the surface.

Fig. 9. The San Andreas Fault in the Carrizo Plains Region.

The classical example of earthquake faulting is the California earthquake of April 18, 1906[1]. At the time of the earthquake a portion of the San Andreas Fault in the coastal region of central and northern California broke for a distance of about 270 miles from San Juan on the south to Upper Mattole on the north (see map, Figure 11). Along the entire break, which was quite clear cut, the coastal side moved northwest relative to the landward side.

The displacement was almost purely horizontal and attained a maximum near the town of Point Reyes Station at the

Courtesy of J. B. Macelwane.

Fig. 10. San Andreas Fault near Logan, Granodiorite to the Left and Tertiary Shales to the Right.

head of Tomales Bay. Here the horizontal relative displacement of the two sides was 21 feet. Figure 12a shows a picture of the road on which this displacement was measured. From this point toward either end of the fault

the observed shifting died out. This earthquake was one of a very few in which it seems beyond doubt from surface

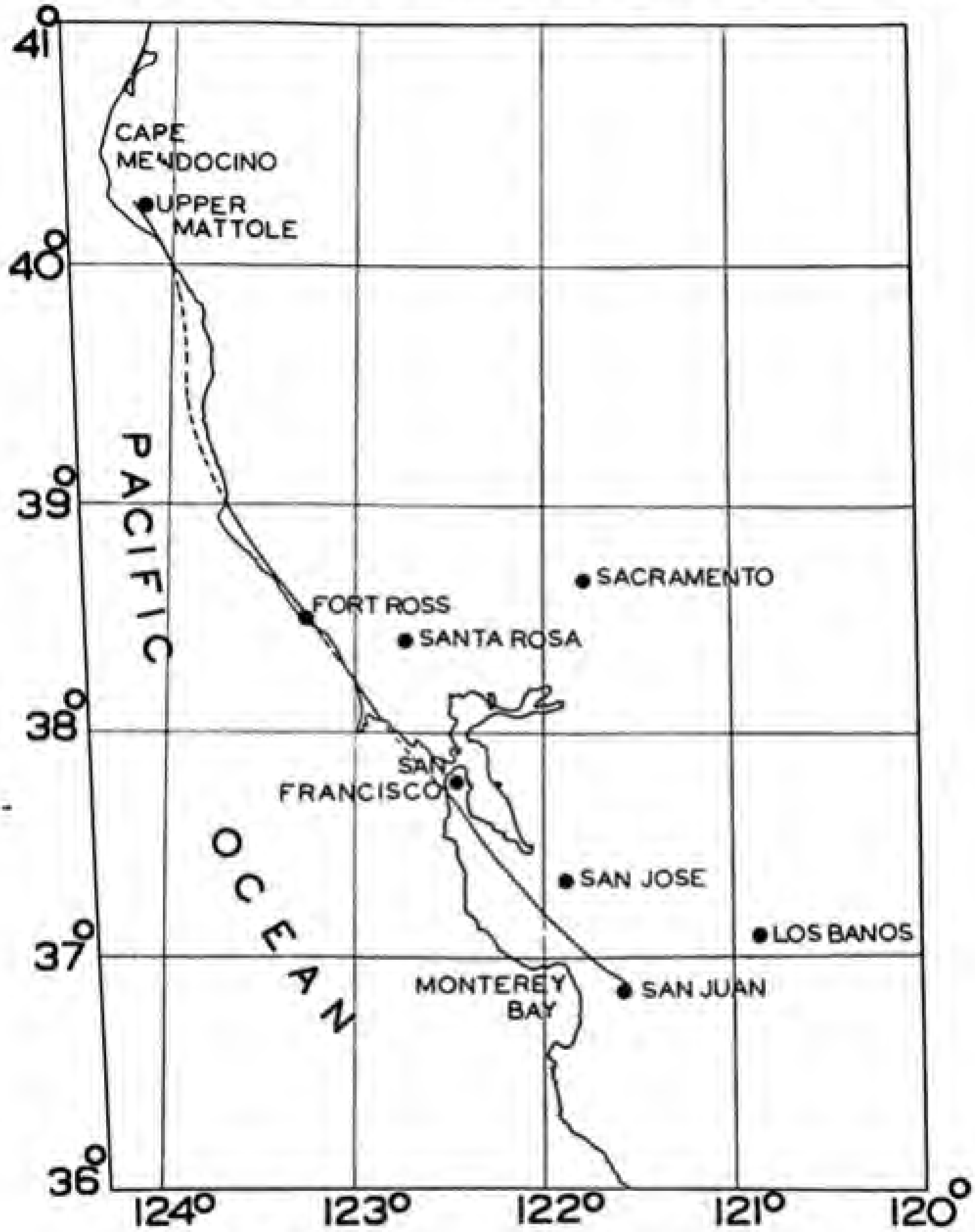


Fig. 11. The Portion of the San Andreas Fault which Broke in 1906.

observations that the faulting was of sufficient magnitude to have caused the earthquake. Some other large shocks accompanied by conspicuous surface faulting are listed in Table 2.

TABLE 2
FAULTING

Locality	*Date*	*Length (mi.)*	*Maximum displacement (ft.)* horizontal	*Maximum displacement (ft.)* vertical
Wellington, New Zealand	1855	90		9
Sonora, Mexico	1887	35		26
Mino-Owari, Japan	1891	40–70	13	20
Baluchistan	1892	"several miles"	2	
Assam, Chedrang Fault	1897	12		35
Formosa	1906	30	8	
California	1906	270	21	
Pleasant Valley, Nevada	1915	20		15
Murchison, New Zealand	1929	more than 2½		15
Imperial Valley, California	1940	at least 40	at least 15	

Considering the great number of earthquakes on record, the number accompanied by marked surface rupture of

Fig. 12a. Road at the Head of Tomales Bay (Looking West) Displaced 21 Feet by the 1906 Earthquake. (From the *Report of the State Earthquake Investigation Committee*, published by the Carnegie Institution of Washington.)

faults is seen to be pitifully few. Some surface cracking and minor faulting throughout the epicentral area is

observed in many more shocks, but in many cases this faulting might well be the result of the shaking and not its cause. Often, as in the case of the great Owens Valley (California) earthquake of 1872, the surface faulting is confined to a long narrow zone but consists of a series of faults more or less *en echelon* on which the displacement is not concordant, being on one fault an upward motion of the

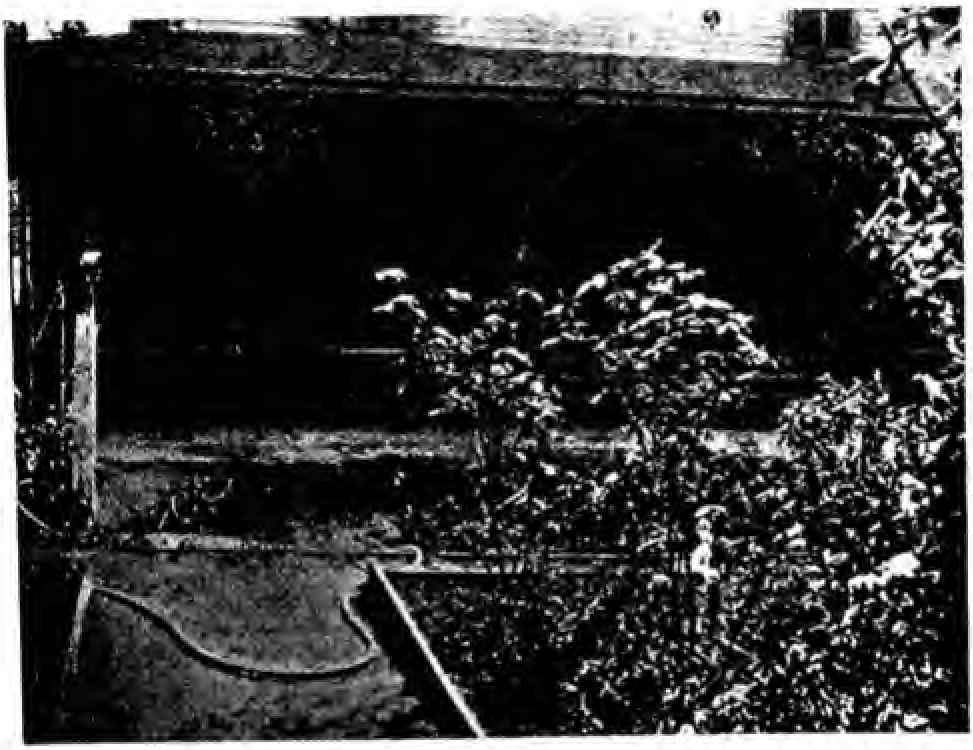
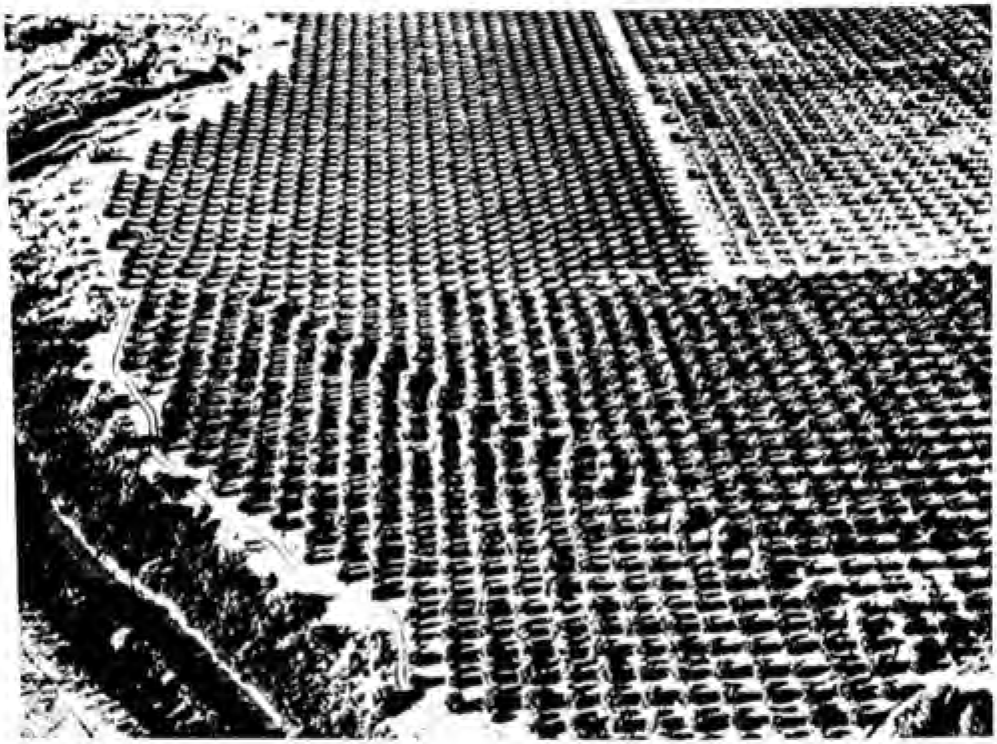


Fig. 12b. Sidewalk and Garden of Bear Valley Dairy Ranch House Displaced 17 Feet by the 1906 Earthquake. (Walk Led Directly to Front Steps before the Earthquake.) (From the *Report of the State Earthquake Investigation Committee*, published by the Carnegie Institution of Washington.)

easterly side and on the next a downward motion of the easterly side. In such cases it would appear that uniform displacements at depth may have resulted in nonuniform displacements at the surface on account of the heterogeneity of the surficial ground. This explanation does not seem wholly satisfactory, however.

Although in general there is the tendency noted above for the epicenters of earthquakes as located from seismograms

to line up along known faults, they do not do so invariably. That they do not may be due to the difficulties in the way of accurately locating epicenters. Again, there may well be faults at depths not yet recognized or recognizable at the surface. Certain it is that not all faults recognizable geologically are today the source of shocks. But the history of the earth's crust is checkered, and many regions quiescent today were certainly active in the past.



Courtesy of Life Magazine

Fig. 13. Displacement of an Orange Grove by the Fault, Imperial Valley Earthquake of 1940.

On the whole it seems reasonable in our present state of knowledge to assign faulting as the cause of all large earthquakes and almost all small ones. Possible exceptions will be discussed later.

Although a large earthquake may exhibit a great fault displacement, as in the California shock of 1906, such are not the only ground displacements accompanying the shock. The other displacements, however, are classed as results of the earthquake and will be discussed in a later chapter. These results may include a certain amount of auxiliary

faulting in rocks adjacent to the main fault. Although a fault break such as that in 1906 or in the Imperial Valley shock of 1940 may be clean cut (see Figures 12 and 13), an examination of the rocks about the 1906 fault shows a wide zone (miles perhaps) in which the rocks are shattered. Presumably then, earlier earthquakes have broken along other parallel lines perhaps as clean cut as those in 1906. It is not necessarily that a number of parallel faults exist at

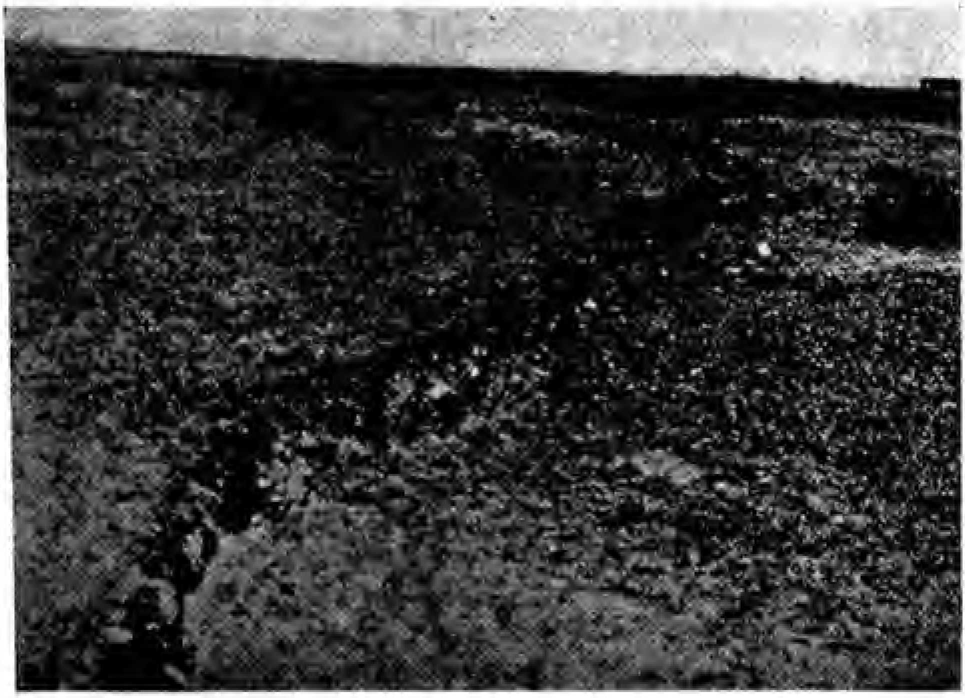


Fig. 14. Faulting in the Utah Earthquake of March 12, 1934.

great depth. It may be that it is only near the surface that heterogeneous conditions cause the fault to have slightly different traces at the times of succeeding shocks.

Elastic rebound theory

At first thought it may seem that faulting should be considered due to some mighty blow struck from below and perhaps be called a result of the earthquake. The problem is then set as to the source of the blow. The only source available would be some sort of explosion similar to those

observed in connection with volcanic phenomena. Faulting in the vicinity of an active volcano may well be assigned to such a cause. But the largest earthquakes do not occur in connection with volcanic activity. With a very few exceptions the earthquakes which are correlated in time and space with volcanic activity are felt in only a very small region. They may be very intense in the immediate vicinity of the crater and be imperceptible ten miles away.

The fault break in California in 1906 was about 270 miles long, and in a region which does not exhibit volcanic activity. No one explosion could cause a break of such length, and to assume a linear series of coincident explosions is foolish. What then could cause such a break? It must have been due to some other type of geologic activity. These processes are in general slow ones. It appears that the energy resulting from some slow, long-continued process must have been stored gradually, to be finally released at the time of the shock. The obvious way of storing such energy is in the elastic strain of the rocks. This strain accumulates slowly, until finally the breaking stress is reached. Rupture, that is, faulting, then occurs. Since repeated earthquakes occur from the same fault, it may be a question of the accumulation of sufficient stress to overcome the static friction on the fault surface rather than the reaching of the breaking stress of fresh rock, since it is not known how much a fault break may "heal" between shocks. Near the surface the fault zone is much fractured and presents an obvious weakness, but at greater depths such zones perhaps do not persist. Indeed at considerable depths the fault may be a sharp surface rather than a zone and may be quite thoroughly cemented between shocks.

The Elastic Rebound Theory postulates a slow accumulation of strain until rupture occurs.[2] All of the energy released at the time of the shock is just before the shock in the form of potential energy of elastic strain in the rocks on either side of the fault. As first stated by Harry Fielding Reid and Andrew C. Lawson the theory was based on

triangulation surveys of the United States Coast and Geodetic Survey made over a period of years before the 1906 earthquake and just after it. The observations were compared, holding stationary a base line from Mt. Diablo to Mt. Mocho, some 33 miles east of the fault. There had been two sets of surveys before the earthquake: one set between 1851 and 1865 and one between 1874 and 1892. Between the first and second sets the Farallon Lighthouse (22 miles west of the San Andreas Fault) had moved 4.6 feet northwesterly roughly parallel to the fault. Mt. Tamalpais (4 miles east of the fault) had moved 5.4 feet in the same general direction. Between the 1874–1892 surveys and the 1906–1907 surveys, after the earthquake, the Farallon Light moved another 5.8 feet northwesterly, this time not so nearly parallel to the fault (about 27° to it). Tamalpais had meanwhile moved back 1.9 feet. There were of course some observational errors in these values, but they were adjusted carefully, and the reality of the displacements is beyond doubt even though their magnitudes and directions contain some error. Thus between the earliest and latest of the surveys in question the Farallon Light had moved a little over ten feet northwesterly in the general direction of the fault. This is attributed to a slow drift of the coastal region north relative to the inland region. Tamalpais participated in the drift up to the time of the shock but slipped back at that time, sudden elastic rebound carrying points near the fault on the easterly side southerly and on the westerly side northerly. A number of triangulation stations in addition to Tamalpais in the region between the Diablo-Mocho line and the Farallon Islands were occupied in the second survey (1874–1892) and reoccupied in the third survey (1906–1907). Table 3 [after Reid[1]] shows how the positions of these had shifted between the two surveys.

The shifts at a number of stations at approximately the same distance from the fault and on the same side of it were averaged and tabulated against that distance. It is seen from the table that, in the interim, points east of the fault

had moved south (relative to the Diablo-Mocho line) while points west of the fault had moved north. Points nearer the fault were displaced more than points farther away. Points west of the fault had been displaced more than points east of it. Figure 15 represents in graph form the data of Table 3. These displacements include any occurring suddenly at the time of the earthquake together with any accumulating slowly between the surveys. The dis-

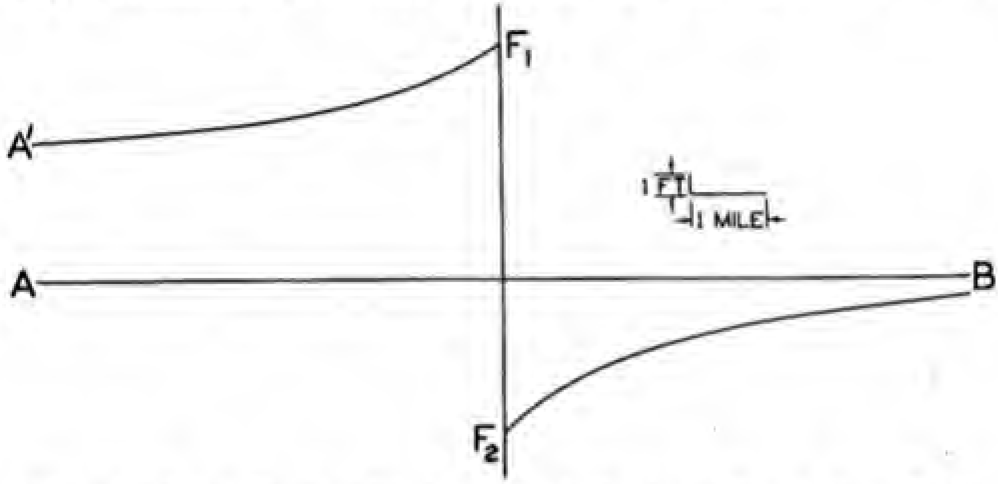


Fig. 15. Graph Showing Displacements in Region of San Andreas Fault.

tance F_1F_2 in the figure is taken as 20 feet, the maximum fault displacement at the time of the earthquake being 21 feet at the head of Tomales Bay.

TABLE 3
AVERAGE DISPLACEMENT OF POINTS BETWEEN SECOND AND THIRD SURVEYS
(RELATIVE TO DIABLO-MOCHO LINE)

NUMBER OF TRIANGULATION POINTS	AVERAGE DISTANCE FROM FAULT		DISPLACEMENT	
	East *miles*	*West* *miles*	*Northerly* *ft.*	*Southerly* *ft.*
1	4.0			1.9
3	2.6			2.8
10	0.9			5.1
12		1.2	9.7	
7		3.6	7.8	
1		23	5.8	

The curvature of the lines in Figure 15 (grossly exaggerated by the scale) must represent elastic distortion in the rock at the time of one of the surveys, since no displacement

of the territory on each side as a whole could cause it, and the assumption of numerous small blocks would demand other faults breaking among them. That the elastic distortion was present just after the earthquake, that is, at the time of the last survey, seems unlikely and is certainly not the case if the elastic rebound theory is accepted and elastic strains assigned as the cause of the shock. Therefore the elastic deformation represented by the curvature of the lines was present at the time of the 1874–1892 surveys. The curvature indicates that by far the greater part of the elastic distortion was concentrated in a region extending less than 10 miles on either side of the fault.

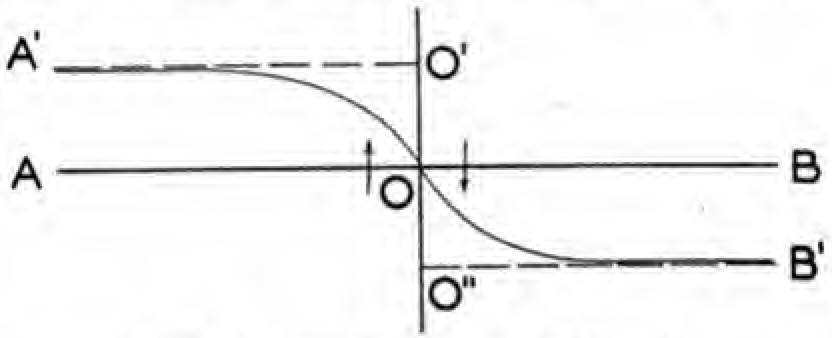


Fig. 16. Illustration of Elastic Rebound.

Since the fault extended for some 270 miles, such deformation could be due only to shear, no bending with components of compression and distension being possible in a beam only 20 miles long and 270 miles thick. Moreover the Farallon Light at a distance of 22 miles west of the fault was well out of the zone of appreciable elastic strain. It follows that the displacements northwestward of the Farallons relative to the Diablo-Mocho line were accumulated by slow drift. These displacements amounted to 4.6 feet plus 5.8 feet, or 10.4 feet between the surveys of 1851–1865 and 1906–1907. Since this is only half the maximum displacement on the fault, half the strain must have already accumulated at the time of the 1851–1865 surveys. Ten feet of strain accumulated in approximately 50 years. This figure should not be taken as a basis of

prediction since it is not certain that strain is accumulated at an even rate, although such seems not improbable.

With the aid of Figure 16 the elastic rebound theory may again be stated. Line AOB is drawn at a time when there is no strain in the earth's crust in the region, presumably just after a great earthquake. The arrows represent a force couple acting on the rocks. These distort the crust elastically until line AOB becomes line $A'OB'$. The curvature of $A'OB'$, drawn from the observations cited above, was attributed by Reid to the action of the couple from *below* the crust, probably by a viscous drag. (In experiments with blocks of jelly, described in the Report of the State Earthquake Investigation Committee, Reid duplicated the survey observations and fault displacement, with the single exception that there was no curvature in the strained lines. His forces were applied from the sides.) When AOB has been strained to $A'OB'$, the rocks have reached their breaking stress (or the stress has reached that value which will overcome the static friction on the fault) and the rupture occurs along $O'O''$. After the earthquake, $A'OB'$ has become $A'O'$ and $O''B'$ and there is no strain in the region. From the great number of aftershocks following a great earthquake it is seen that this condition of no strain is not attained by the main shock alone. Some Japanese investigators have concluded from the nature of the first motion of the aftershocks as compared to that of the main shock that faults may sometimes fling beyond rest during the main shock and recover in aftershocks. Gutenberg's studies in southern California indicate that aftershocks are caused by movement in the same direction as the main shock.

According to the elastic rebound theory the immediate source of the energy sent out as elastic waves in an earthquake and causing the faulting is elastic energy of strain accumulated slowly in the neighborhood of the fault. It is the grating of the fault surfaces against each other that generates the elastic waves. One may hold two files in his hands and rub them together. He feels then the resulting elastic vibrations as they pass through his hands.

Other theories

It has been emphasized above that only a very small number of the earthquakes felt and recorded by instruments are accompanied by visible faulting at the earth's surface. And of those accompanied by faulting the types of breaks at the surface are often divergent in character in adjacent localities. The generally accepted cause of most earthquakes is nevertheless faulting, even though this faulting must be postulated as occurring at some depth with no surface manifestation. It is obvious that theories other than those which postulate faulting may be advanced as to the cause of earthquakes not accompanied by clean-cut surface faulting and that they cannot *a priori* be labeled as undesirable. If these theories attempt to attribute all shocks to some other cause and postulate that faulting is secondary and a *result* of the earthquake, then they can be criticized on the basis of the 1906 earthquake observations. In this case, and in some others, the surface rupture was of such an extent as to indicate that the faulting was of sufficient magnitude to have been the source of the energy in the earthquake waves. Reid[1] computed 1.3×10^{17} foot pounds as the energy probably released at the fault. Now if there is available a visible possible source of sufficient magnitude to have caused the shock, it would be poor scientific procedure to assume that this source is also secondary and to postulate some uncertain phenomenon at depth as the cause of both earthquake waves and faulting. Again, if the new theory attempts to explain only those earthquakes which do not exhibit clear surface faulting, it faces criticism as dubious scientific procedure because it adds a new kind of source tending to complicate rather than simplify the science. Such complication necessarily arises on occasion in the history of any science when new data demand it, but should not be advocated unless observations rigidly require it.

However, other theories have been advanced and should be considered with care. R. D. Oldham for many years strongly advocated the theory that faulting was very fre-

quently a secondary phenomenon. He postulated some considerable change in volume of large masses at a depth in the earth measured in hundreds of kilometers. Such a change he considered as the source of all large earthquakes. The change was to take place with explosive violence and to produce two results: elastic waves sent out all over the earth and faulting near the surface, which in itself would be a large source of damaging vibrations in the epicentral region. For the action at the deep source he proposed the name *bathyseism;* for that near the surface, *engyseism.* His argument was based largely on very uncertain instrumental location of focal depths compared with rough computation of the focal depth of the same shock based on field data. Today seismologists would doubt very much the calculations based on the instrumental data and also feel considerable lack of confidence in the methods of computing depth of focus from the field observations.

In recent years some Japanese seismologists headed by M. Ishimoto[3] have advocated the theory that most earthquakes are due not to faulting but to movements of magma at depth. Cavities filled with magma (molten or plastic rock) are postulated, not necessarily at great depth in the earth. Chemical and physical changes in these magma reservoirs result in comparatively slow changes in pressure within them. If the pressure rises high enough, some of the magma is violently ejected from the chamber into that portion of the neighboring rock which offers least resistance. It is this sudden movement of magma which is the source of the earthquake waves. Changes in level in various parts of Japan both before and after large earthquakes are cited as evidence of pressure changes at depth.

Efforts to correlate earthquake occurrence with weather and positions of heavenly bodies have been numerous since the beginnings of earthquake knowledge. None of these has been sufficiently successful to warrant more than mention here. No scientist in studying such correlations considers that these phenomena might be the principal

cause of earthquakes. He envisages a slow accumulation of strain in the earth's crust with the possibility that the small, additional stresses due to atmospheric pressure or gravitational attraction might serve as *trigger* pressure to release the shock.

There seems always to be a legion of nonscientists who are attempting to establish as direct causes of earthquakes some activity in the heavens. They attempt this without first mastering what is already known about earthquakes and astronomy. They are woefully disgruntled when they cannot immediately gain the ear of the scientific world and when the press has finished giving them their week of notoriety. They remind one of a man who goes to a bridge tournament with a set of rules of his own and a brand new method of keeping score. Such men should play bridge at home with the family. Unfortunately the family is rarely interested either.

The earthquakes discussed above are called *tectonic*. Tectonic processes are those connected with the major processes of mountain building and earth evolution. Very little is known certainly about the exact nature of the fundamental underlying causes, as will be seen from the next chapter.

Volcanic earthquakes

In those cases in which miniature earthquakes occur during or immediately before or after volcanic eruptions, the underlying cause of minor faulting in the vicinity of the crater is rather obvious. Here we need not postulate any very slow accumulation of strain, for the movements of volcanic gases and molten masses are often rapid. These break through to the surface, and we are assured of their presence. No doubt some small volcanic earthquakes are due directly to explosions which may not necessarily even rupture the walls of the cavity in which they occur. However, it seems likely that most volcanic earthquakes are caused immediately by faulting phenomena on a minor

scale, with the underlying cause of faulting known. In a region of quiescent volcanic activity small earthquakes continue to occur in large numbers, and these too may be called *volcanic earthquakes,* although in such cases movements of magma and gases are not visible but inferred.

Thus the distinction between tectonic and volcanic earthquakes is not a good one; it does not differentiate as to immediate cause; it merely infers that for the former there is no obvious underlying cause while for the latter there is. Certain distinctions between the two types are possible only at the time of a volcanic eruption, for it might be argued with some plausibility that earthquakes occurring at other times in a volcanic region are caused by slowly acting tectonic forces as distinguished from sudden movements of volcanic magma and gases.

Volcanic earthquakes are rarely if ever large shocks from the point of view of total energy released. Their small size seems reasonable, since movements of magma bodies and gases at the time of volcanic eruptions are local rather than regional phenomena. A volcanic earthquake may do great damage on the flanks of the volcano and be scarcely recorded on a seismometer one hundred miles away. Such a shock centers very close to the surface of the earth, and the energy in the vicinity of the epicenter is not much dissipated by spreading out. There are on record very few large earthquakes correlated closely in time and space with volcanic eruptions. Davison[4] cites the earthquake of January 12, 1914, which occurred a few hours after the great eruption of Sakura-jima in southern Japan. This earthquake damaged houses in the neighborhood of the volcano and was strong enough to be recorded on seismographs in Europe. On February 13 of the same year a similar earthquake accompanied an eruption of Iwojima, some fifty miles south of the former epicenter. On April 2, 1868, a great earthquake occurred in the southeastern part of Hawaii. It accompanied an eruption of Kilauea. A fault 1½ kilometers long broke, and lava flowed out of it.

Stone houses were destroyed. The shock was strong enough to rattle windows and doors 190 kilometers from its center. These three exceptions to the general rule of smallness for "volcanic" shocks merely lead to the classification of these three shocks as tectonic. They were too big to allow an easy inference that they were due to magma movements beneath the volcano rather than to regional strains in the crust.

In general, volcanic earthquakes are more likely to occur before eruptions than after them. According to Imamura[5], they occur in series with increasing intensity before an eruption, with very few after it. In contrast, large tectonic shocks always have a long series of aftershocks of diminishing intensity, whereas foreshocks are few or lacking.

Collapse earthquakes

The older classifications of earthquakes included *collapse* earthquakes. Early writers attributed very many shocks to the collapse of great underground caverns. Since the existence of these caverns was assumed in order to explain the shocks, the scientific procedure was not very sound. (In attributing earthquakes to faulting, the faults are usually visible at the surface, even though they do not rupture at the surface very often.) Modern writers tend to drop the classification of collapse earthquakes. However, in regions of limestone caverns and in regions of abandoned coal mines the collapse of one of these openings sometimes produces a small earthquake immediately above it. Sieberg[6] lists as the regions having the greatest number of collapse earthquakes Luzon, Tabasco, Yucatan, Cuba.

There is a popular impression that the removal of gas and oil from a district might well cause earthquakes. But when gas and oil are removed from the earth, water immediately takes their place. However, Sellards[7] has pointed out that this replacement is accompanied by a reduced pressure and that in the regions of some oil fields subsidence of the surface

has accompanied removal of oil and gas. He cites the Goose Creek oil field in Texas, where subsidence of as much as three feet has been observed. He suggests that small earthquakes might result from rupture of incompetent underground structure in such a case and cites as a possible example the Wortham-Mexia (Texas) earthquake of April 9, 1932. He points out, however, that there are known faults in the region.

It appears that faulting must be the immediate cause of most of the earthquake waves even in collapse earthquakes. The rupture of the strata at the time of collapse and not the impact of the falling roof seems the more likely source of most of the waves, since a collapse such as this is not like a free fall of a body.

The old subdivision of collapse shocks which has suffered most in modern opinion is that of earthquakes caused by landslides. It does not seem likely that any landslide would proceed rapidly enough or stop sufficiently quickly to give an earthquake-producing impact to the ground. Such slides proceed relatively slowly, the potential energy in the mass before rupture being largely dissipated in frictional heat, the mass crushing a zone of rock on which it slides. Of course such progress will produce some vibrations, owing to its faulting characteristics, but probably not sufficient to justify its being given a classification as a cause of earthquake. Macelwane discredits landslides as cause of earthquakes, citing a number of large landslides which have taken place without recorded shocks. The classic example of landslide shock cited in the past was that of February 18, 1911, in the Pamirs. A great earthquake recorded in Europe centered in the general region in which a mountain had slid down into a valley, blocking a stream and forming a lake. The exact time of the landslide was not known even to the nearest day. It was for a time popular to compute the energy in the earthquake from seismograms and compare it with the potential energy lost in the slide. Different authorities computed different

energies, but thought that that of the landslide and that of the shock were of the same order. Of late the general opinion seems to be that it is better to class the landslide as the result rather than the cause. The impact of the great meteor of June 30, 1908, near Vanovara, Siberia, has been correlated by some writers with an earthquake recorded at Jena in Germany. This has been discredited by Macelwane because no great shock was reported from that part of Siberia.

Deep focus earthquakes

It has been established definitely since Wadati's great work in 1928 and later that some earthquakes occur at depths as great as 700 kilometers below the surface of the earth. Some lines of evidence appear to indicate that at depths greater than, say, 100 kilometers the rocks have no strength or very small strength. These lines of evidence are discussed in the next chapter. The rocks at these depths do have large rigidity, however, since in them S waves (shear waves) are transmitted without difficulty. The question which arises is: "Does the existence of earthquake foci at depths between, say, 100 kilometers and 700 kilometers demand that the rocks in this region have considerable strength as well as rigidity?"

Now a lack of strength of the rocks at these depths merely means that if small additional stresses are put upon them, they will flow plastically *if given time enough*. Certainly an explosion occurring in their midst would be a source of elastic waves and would result in an earthquake. But the first P motion to record at all stations should in such a case be a compression. This is not the case. The nature of first motion of deep focus earthquakes at various stations is distributed between compressions and rarefactions, as is the case for shocks of shallow focus. As far as seismograms can indicate, the behavior at the source of the shock is similar in the case of deep focus shocks and shallow focus shocks. This similarity would indicate deep seated fault-

ing. Some writers have objected to faulting in a medium of little strength, saying that no strain could accumulate there. The weakness of this objection lies in its ignoring the *time* factor. Strain may accumulate if it works fast enough. A bar of wax clamped at one end in horizontal position will, if given sufficient time, bend down of its own weight. It has insufficient strength to support its own weight. But if one adds weights to its free end, it will break when the load becomes large enough. The loads have been added faster than the plastic flow of the wax may accommodate them. Perhaps faulting can occur at depth in rocks which will flow if given time enough.

However, laboratory experiments on rocks at high pressures have not indicated a tendency for strength to decrease. But these experiments have not yet included high temperatures, which no doubt have considerable effect in the earth. To find a possible underlying cause of a rapid accumulation of strain at depth is troublesome.

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CHAPTER IV

Underlying Causes of Earthquakes

Introduction

We have assigned faulting as the most probable cause of earthquakes, and, more than that, faulting due to strain that has accumulated slowly. Such a cause is not very satisfactory unless we can find sufficient possible sources for a slow accumulation of strain.

It is not necessary, not even desirable, considering the complexities of the earth's surface history, that only one possible source be selected. When advancing new theoretical explanations of the form of the earth's surface many students of earth history seek first to destroy all previous concepts. This seems unwise. The earth's surface has apparently in the past been subjected to spasmodic upheavals sufficiently irregular in place and time to suggest strongly the action of varied phenomena within the earth.

Therefore, in considering the possible forces discussed below, together with arguments for and against each, let us not feel that we must espouse one and condemn the others, nor indeed that we should condemn all because no one of them seems to account for the varied history of the crust.

Contraction of the earth

Most theories of the origin of the earth assume that it has cooled from a molten mass. It is now solid, that is, possesses rigidity, at least as deep as its core at depth 2,900 kilometers, as is shown by the free transmission of shear waves to this depth. (The radius of the earth is about 6,370 kilometers.) Cooling of a mass as large as the earth is a slow process. At successively greater depths in the

earth the rocks have cooled by less and less amounts since solidification. The amount of heat the surface receives by radiation from the sun is so vastly greater than the amount received by conduction from below that the latter is negligible. The surface is therefore in radiation equilibrium with the sun. The surficial layer of the earth is not changing in temperature and so not changing in volume. It tends to become too large to fit the cooling and shrinking layers below it and must collapse to shorten the circumference. In this behavior is one very probable source of the accumulation of elastic strain in the crust, with resultant folding, faulting, and earthquakes. These processes build mountains. It is debatable whether or not shrinking of the earth has been capable of producing all the crustal shortening observed by geologists. Certainly it is not the only factor in deforming the earth's crust. But it remains, in the author's opinion, the most likely of all the underlying forces upon which call can be made to explain the slow accumulation of strain required by the elastic rebound theory of faulting. The theory of the shrinking earth has the added advantage that it calls for recurrent periods of mountain building with times of relative quiescence between them. Collapse would not occur until the slowly mounting strain reached the yield point of the surface layer. Collapse would then relieve the strain, and a period of relative quiet would follow until the yield point was again attained. From geologic evidence it is known that periods of intense diastrophism and mountain building have been intermittent.

Whether or not crustal rumpling due to shrinking might be distributed as irregularly over the surface as our areas of mountain building indicate is a moot question. Opponents of the theory that crustal shortening is the main source of diastrophism claim that the effects of such shortening would be more wide spread than is observed.

Again there is the question: How much of the crustal shortening inferred on geological grounds may be explained by the theory of a shrinking earth? Jeffreys has computed

the crustal shortening caused by cooling as about 50 kilometers per period of collapse. That inferred by many geologists from folding and thrusting in various mountainous regions of the earth is much greater. Jeffreys would attribute much of this shortening to collapse and sliding of mountains raised too high to support their own weight, rather than to true crustal shortening.

The speed of rotation of the earth is apparently slowing. The rate of secular acceleration of the moon's motion indicates that some 1,600 million years ago the earth day was only 0.84 of its present length. The result of such decrease in the angular velocity of the earth would be a decrease in ellipticity and in volume. These effects, according to Jeffreys, would have had their major effect early in the history of the earth and can hardly be called upon to explain present-day mountains and earthquakes.

Isostasy

We know that rocks at the surface of the earth have strength, for we have mountains and we even use the rocks for buildings and other structures. However, we know from common experience that substances lose their strength at high temperatures. That the temperature of rocks increases with depth in the earth's crust is attested by observations in mines and bore holes. The mean value for sedimentary areas in the United States lies between 15° and 30° C. per kilometer[1]. Although it appears improbable that this rate is maintained to great depths, it is apparent that the temperature is considerably higher below the crust than in it, and a failure of strength in rocks below the crust would not be surprising, although it is not known as yet how far the effect of higher temperature toward lowering the strength is counteracted by the effect of the high pressures toward increasing the strength.

About two hundred years ago Bougier observed in the Andes that the mountain mass of Chimborazo in Peru did not attract a suspended pendulum by as great a force as it

should if the mountain were merely an added mass on the earth's surface[2]. Similar experiments of the deflection of the plumb bob by mountain masses in various parts of the earth have regularly shown the same phenomenon. Since the densities of the rocks which compose mountains are quite well known and the distribution of the mass of the mountain directly observable, the force of attraction exerted by the mountain alone may be computed by the inverse square law of attraction. If a pendulum is suspended in a valley near the mountain, such an attraction compounded with normal gravity should deflect the pendulum by a computable amount. The actual deflection may be obtained by setting up pendulums successively on two opposite sides of the mountain and observing in each case the acute angle made by the plumb bob with the direction of the line of sight to a distant star. These angles are made up of two parts, the one due to deflection by normal gravity and tending to make the bob point approximately to the earth's center and the other due to deflection by the attraction of the mountain mass. The sum of the two former angles is known, being equal to the angle subtended at the earth's center by the arc of the earth's surface between the two observation points. Therefore, the sum of the two latter angles, those due to attraction of the mountain, may be computed. It is found regularly that these angles are much less than those computed from the known mass and distribution of the mountain.

This difference must be explained by a deficiency of mass in the material under the mountain and leads to the theory of isostasy, which states that any excess of matter on the earth's surface (such as a mountain) above a standard level is balanced by a defect below it, whereas any defect of material on the surface (such as an ocean depth) below the standard level must be balanced by an excess of mass below it. The net effect is that at some depth below the earth's surface (probably less than 100 kilometers) there is a surface above which the weight of all columns of the earth's crust

is the same, regardless of whether they terminate upwards in great mountains or in deep seas.

How this compensation of mass might be accomplished has been explained in two ways. Airy suggested that the upper crust is made of lighter rocks than the layer below it and that the light rocks effectively float in the heavier material below, high mountains having deep roots. This conception is frequently called the "roots of mountains" theory. As a mountain is gradually eroded away, its material being deposited in near-by depressions, there is a flow of the heavier material below in the reverse direction, keeping the total of mass the same in each column above the level of isostatic compensation. Such a process would slow down the progress of base leveling very much, although it would eventually be brought about, since the volume of the heavy rock introduced under the mountain by undertow would always be less than the volume of the equal mass of light rocks removed from above.

Pratt evolved the alternative explanation requiring mountain columns to be of less density than oceanic columns all the way down to the level of isostatic compensation. This concept seems unable to deal with erosion and deposition, for only undertow seems physically capable of causing retention of the balance. The Airy concept meets some difficulties in regions in which geological record indicates failure of the surface rise where folding of light surface rocks has thickened the surface layers. Reid cites cases in which such areas have even been depressed for long periods after folding, only to be uplifted at a much later date. It is very clear that if isostasy is at work it is not by any means the only force that moves the crust.

Direct measurements of gravity confirm the conclusions drawn from the deviation of the vertical that mountains are not added masses on the earth's surface. The test of any theory as to the distribution of mass within the crust is the smallness of the gravitational anomalies computed on the basis of that theory. An anomaly is the difference

between the observed value and the computed value (observed minus computed). Two gravitational anomalies frequently computed are the free-air anomaly and the Bougier anomaly. The free-air hypothesis effectively assumes that mountains are hollow and computes gravity the same at a given height whether from a balloon over the ocean or on a mountain. The Bougier anomaly is computed by regarding a mountain as an added mass on the earth's crust with no compensation beneath it. It has been found that the free air anomalies are usually much less than the Bougier anomalies. The isostatic anomaly depends on the nature of the assumptions made regarding compensation. The United States Coast and Geodetic Survey has been able to reduce anomalies throughout the United States by assuming uniform compensation (Pratt isostasy) and a depth of compensation of some 60 to 100 kilometers. Heiskanen in Europe has been equally successful using the Airy idea of isostasy in which mountains are compensated by roots. His average layer thickness was about 40 kilometers. In the East Indies and India, two other regions which have been surveyed for gravity, the isostatic anomalies are not small. On only a very small portion of the earth's surface have systematic gravity observations been made, and over considerable parts of even this small region the data do not indicate isostatic adjustment.

Any tendency for the maintenance of gravitational balance in the earth's crust would offer a source for that slow accumulation of strain required by the elastic rebound theory of the cause of earthquakes.

Drifting continents

The general idea of the permanency of continents and oceans is widely accepted in geology. The lack of deep sea sediments on continents is evidence of such permanency. Although sedimentary rocks laid down in shallow seas are common, there is an obvious lack of evidence of deposits characteristic of the deep ocean bottom. The general

prevalence over the earth's surface of two predominant levels, one a deep sea level and the other the continental surface (near sea level) level, has been cited as evidence of the fundamental difference between continents and oceans. It is true that detailed soundings of the ocean bottom are increasingly indicating a complexity of topography on the ocean floor, once thought comparatively smooth. More details of this character regarding the deeper ocean bottoms may eventually force a reconsideration of the fundamental hypothesis of permanency. The idea that the continents, although relatively permanent, may not always have occupied the same positions on the earth's surface is not very new. The principle exponent of shifting continents, however, was Alfred Wegener[3]. Proceeding from the observation that if South America were moved easterly against Africa, the eastern coast line of the former would fit fairly well the western coast line of the latter, Wegener constructed his hypothesis that all the land masses of the earth were grouped together in one great continent up to Cretaceous time. Then rupture began, South America breaking away from Africa, the rift extending northward until North America and Greenland separated from Europe in the Quaternary. Antarctica, Australia, and India adjoined Africa until the Jurassic, when they began to split away. India in its movement folded the continental mass between it and present Central Asia into the Himalayan highlands. The theory calls for two kinds of rocks near the earth's surface, the competent continental rocks or *sial* (silicon-aluminium) and incompetent oceanic rocks or *sima* (silicon-magnesium). The former, although less dense and less rigid (as known from the speed of S waves) than the latter, are still stronger, the latter failing under small forces over a long time. The sialic continents float in the sima and drift in it. On the forward side of the drifting continents great mountains are pushed up by the forces brought into play, for example the American Cordillera. If such behavior can be granted, there is here an ample

source for the slow accumulation of strain in many of the active earthquake regions of the world.

A great weakness of the theory lies in the lack of any known forces which could promote such drift. Those called on were the equatorward force of Eötvos and the westward force of tidal friction. The former force is due to the ellipticity of the earth. The equatorial bulge causes a slight change in the direction of gravity at successive levels within and above the earth. If a continent is floating in sima, its center of gravity will be higher than its center of buoyancy (the center of gravity of the displaced sima). Now the direction of gravity at the center of mass of the continent will be slightly more equatorward than the direction of gravity at the center of buoyancy. Thus the bearing-up force of the sima will not be directed quite in line with the bearing-down force of the continent, and the resultant will be a slight force tending to drift a continent toward the equator. The magnitude of the force is at its maximum at latitude 45° and is there only about 10^{-6} that of gravity. A sima which could not resist the force that a continent would offer on this account would be unable to support irregularities on its surface of more than a few meters in height. A small westward force exists because of tidal friction, the force which slows the earth's rotation. Its magnitude is only about 10^{-7} that of the equatorward force. Both of these forces seem pitifully inadequate to produce any continental shift.

Gutenberg[4] has modified Wegener's theory by suggesting that the sialic material of the continents has spread by plastic flow rather than by rupture. His idea requires a thin layer of sial under the Atlantic and Indian Oceans. He hints that the source of energy necessary for the spreading of sial may come from heat in the interior of the earth but mentions also the natural tendency for the sial to spread to reduce its gravitational potential energy.

Whether or not the continents are separating may now be tested with accurate methods of determining longitude by

use of radio time signals. To date no conclusive evidence has been shown, but recent exact methods have not been long employed, and the next decade should give evidence one way or the other. Observations of longitude made intermittently on Sabine Island (off Greenland) since 1823 indicate an increase in longitude there, but in no case have the data been sufficiently accurate to establish drift without cavil[5].

Radioactivity and convection currents

Joly[6] has advanced a theory of cycles to explain the pulsations of the earth's surface history. He envisaged the continents as radioactive blankets causing a feebler radioactive sima to melt under them. As the sima melts, the continents sink slowly lower and epicontinental seas invade the margins. For not so clear a reason the sima under ocean bottoms also melts. The upper crust of the earth becomes too small, fissures open, and flows of basic rock occur. When the subcrust is well melted, the small forces mentioned by Wegener or others cause the continents to drift, leaving exposed to the ocean floor the sima formerly blanketed. This now cools. For some unexplained reason the molten sima now covered by the continent in its new position also solidifies. The continent rises as the material in which it floats becomes denser. The earth's crust becomes too large to fit the shrinking subcrust and is folded and faulted during a period of mountain building. This hypothesis has been subjected to many detailed objections as well as to the general objection that in an isolated system such as the earth temperature should tend toward a steady state and not be subject to cyclic alternations.

Holmes[7] and later Griggs[8] have called on convection currents (plastic flow) in the solid earth to explain the forces which cause strain to accumulate in the strong crust, with resultant folding and faulting. Holmes calls on the ellipticity of the earth to produce thicker outer shells near the equator and therefore more radioactive heating. Con-

vection currents in a glassy substratum cause a drag poleward on the outer crust. Similarly, excessive heating and blanketing near the centers of continents cause convection currents toward the coasts, with a resultant tendency to rupture the continents. In some way these convection currents are met by opposed currents coming from oceanward at the continental boundary. The meeting of these two, which then plunge down, compresses the continental borders, throwing up mountain ranges.

Griggs is not content with currents merely in the substratum but on a grander scale requires cyclical circulation in large portions of the earth from core to upper rind. This theory requires that the changes in speed of seismic waves between the crustal layers and the core be due to polymorphic changes of physical state or pressure-induced changes in the elastic constants of a material of common chemical constitution rather than a change in the chemical nature of the rock. There is some difficulty in getting the differential heating necessary to start the cycles. This theory faces the general criticism of cyclical changes of temperature in the earth.

Pekeris and also Vening Meinesz have called on convection currents in the upper 1,200 kilometers of the earth to produce folding of surface rocks.

Undation theory

The Undation Theory of Van Bemmelen[9] calls upon thermal changes in the earth's crust and just below it to bring about magmatic differentiation at depth where the rock is weak. Sial resulting from the differentiation rises, owing to its low density, arching the crust above it. The inflow of sima behind it causes depressions to form on either side of the uplift. Thus we have anticlines and synclines with all the resultant forces in the crust. Elevations of too great height rupture and slide. Such a process offers abundant causes of accumulation of strain in the crust. Objections of course may be offered, as to any other

such general theory yet offered. There is question as to the reasonableness of such very widespread differentiation as Van Bemmelen uses to uplift whole continents. Why should depressions be left by sima flowing under rising sial? Why does not the sima behind it flow with it to keep the general level?

Conclusion

We have considered above a number of suggested causes for forces in the earth's crust that might result in the slow accumulation of strain. Gravitational energy and heat have been called upon. We have excluded direct use of forces exerted by heavenly bodies. There are a considerable number of laymen who wish to assign the cause of earthquakes to the positions of heavenly bodies. The forces set up in the earth's crust by the variation of gravitational attraction of such bodies are small compared to those necessary to cause earthquakes. And the gravitational attraction of the moon is so much greater than that of other bodies that if trigger action may be exerted by such forces then it is the moon which should be the active source. Maxwell Allen made detailed studies of the correlation of the times of occurrence of shocks and the position of the moon. Although he found some correlation, it was sufficiently slight so as to offer no possibilities in the way of prediction. Trigger action which would be repeated as often as the lunar cycle would not be very valuable for prediction even if correlation were close, for large earthquakes occur only once in very many years in a given locality, whereas the trigger force would be a maximum once or twice in each lunar month. Stetson has found some correlation between deep focus shocks and lunar positions. Many other investigators have studied the problem of correlation of the times of occurrence of earthquakes and the position of the moon, but this study has not led and is not expected to lead to earthquake prediction.

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CHAPTER V

Effects of Earthquakes

Introduction

A complete description of an earthquake can never be made from the study of the records of seismographs. In this day of the extensive use of instruments there is grave danger that field observation will be neglected. What happened to the earth's surface features, rocks, and soil and what happened to buildings and other works of man is an object for study of prime importance. Many statements are made to the effect that buildings vary so much as to quality of design and construction and as to geologic foundation that the observation of damage to structures is almost useless as a measure of "intensity" (Figure 17). However, for purely descriptive purposes the distribution of damage should be on record, regardless of whether that damage was due to poor design, poor construction, or poor foundations. After all, one California town probably has about the same types of construction, good, bad, and indifferent, as does another, and observation of what happened to one during an earthquake gives an idea of what may happen to another. It is the town as it has been built and not as it should have been built that will feel the shock (Figures 18a & 18b).

To get a complete record of how the earth shook in a town by instrumental means would require a battery of seismographs in every district of the town. A battery is necessary, since each instrument magnifies the motion in a certain frequency range to the effective exclusion of other frequencies. True, integration of the seismograms will sometimes bring out underlying frequencies not readily evident on the separate records, but integration is a tedious

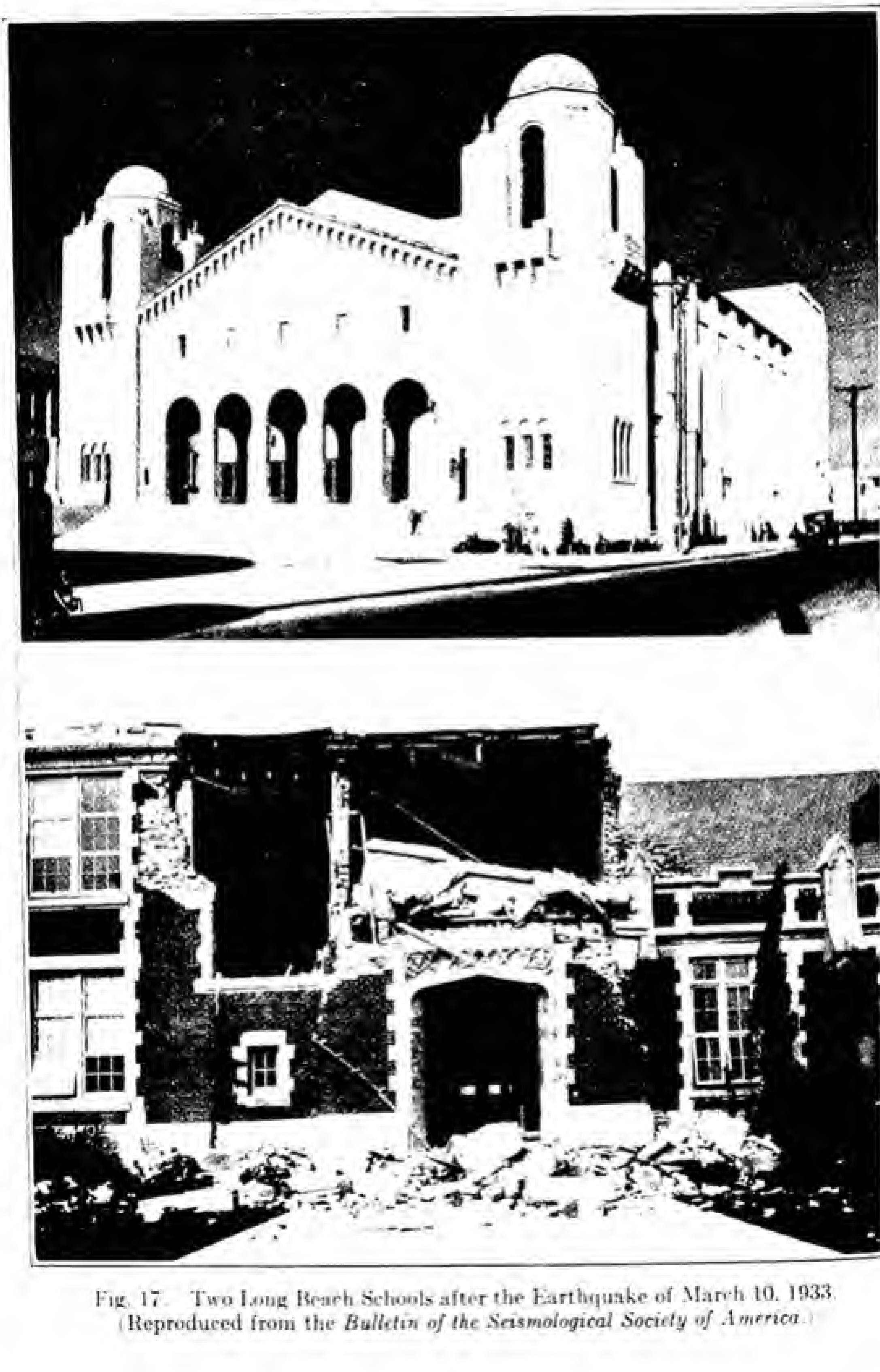

Fig. 17. Two Long Beach Schools after the Earthquake of March 10, 1933. (Reproduced from the *Bulletin of the Seismological Society of America*.)

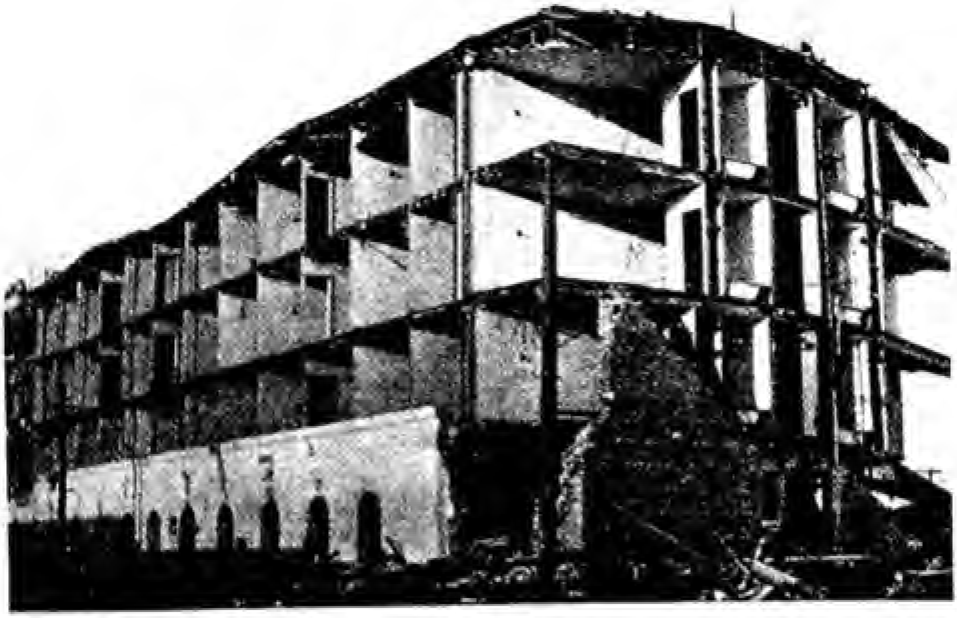

Fig. 18a. A Hotel in Santa Barbara after the Earthquake of June 29, 1925

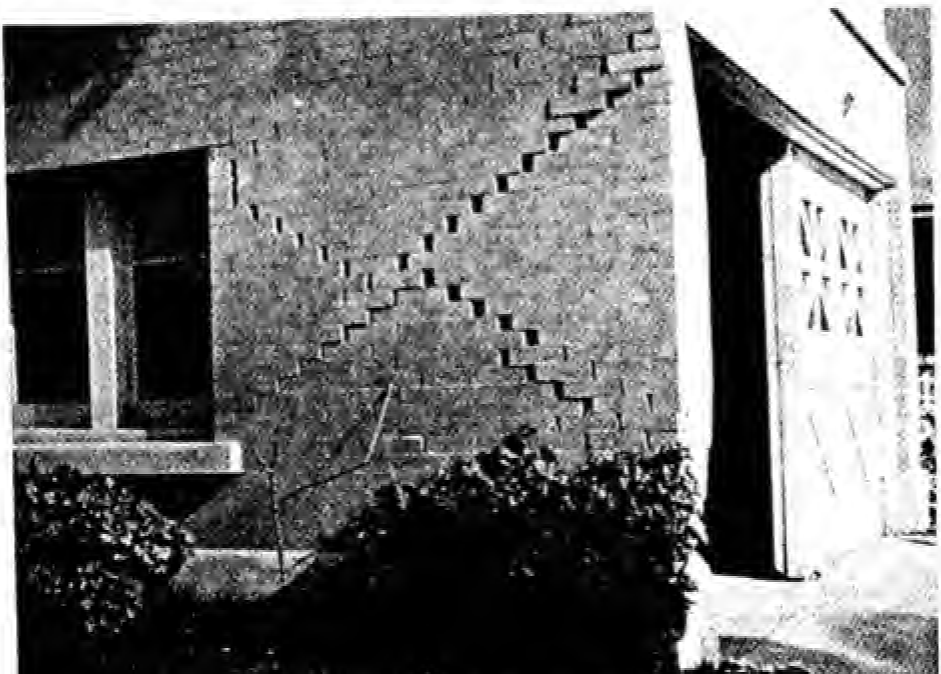

Fig. 18b. A Garage in Long Beach after the Earthquake of March 10, 1933

procedure and subject to error. Even the battery of instruments will tell only of the movements directly under it. And there is complaint now from some engineers that seismographs in a building may be influenced by free vibrations of that building or by those of adjacent buildings. It would appear that what many practicing engineers would like to know is, "How would this locality have shaken if there had been no city here?" This question appears highly academic to the writer, and seismologists cannot answer it. Why not be interested in what happens to buildings as they are in the cities which are here? At the same time, of course, there must be observed by instrumental means in selected localities both in and outside of buildings the exact motion of the earth as nearly as it can be recorded. Similar motion can be imposed on shaking tables on which are models of buildings of various designs, as a test as to their strength to resist. But in this instrumental and laboratory process there are many necessary assumptions and simplifications. Hence the performance of this type of experiment does not alter the great need for observation in the field of what did happen to buildings, even though in most cases exact knowledge of the inner construction of the building and of the earth motion may be unobtainable.

Intensity scales

The *intensity* of an earthquake is a measure of the damage done to structures built by man and changes in the earth's surface, such as fissures in rock and disturbance of soil. Clearly the intensity so defined depends on the stability of the construction by man and on the stability of the earth's surface. A given earthquake is less intense on rock than on swampy fill. It is less intense in a well-constructed building than in a flimsy one. This definition of intensity is very distressing to many "practical" men. They demand some definition of intensity which would be independent of construction and foundation. Professor Richter[1] has suggested

such a measure and called it the "magnitude" of the earthquake. He defines it as the logarithm of the maximum amplitude recorded on a standard Wood-Anderson seismograph 100 kilometers from the epicenter. This gives a number which, by methods of interpolation and extrapolation devised by its originator, can be applied to any well-recorded shock regardless of whether its center lay in a settled region. On this scale the smallest shocks reported felt have a magnitude of $1\frac{1}{2}$, while the largest shocks on record have a magnitude of about $8\frac{1}{2}$. The magnitude of the Long Beach earthquake of 1933 was $6\frac{1}{4}$, while that of the San Francisco shock of 1906 was about $8\frac{1}{4}$. While it is true that the amplitude on a Wood-Anderson seismometer depends on the geologic foundation under it, seismographs are founded on rock, wherever possible, and the Richter number may be evaluated by use of records from several stations.

Intensity scales have been built up from many years of experience in the observation of field data. A typical scale, and the one most used in the United States at present, is the Modified Mercalli Scale of Wood and Neumann.[2] In its abridged form it reads as follows:

MODIFIED MERCALLI INTENSITY OF 1931 (ABRIDGED)

I. Not felt except by a very few under especially favorable circumstances.

II. Felt only by a few persons at rest, especially on upper floors of buildings. Delicately suspended objects may swing.

III. Felt quite noticeably indoors, especially on upper floors of buildings, but many people do not recognize it as an earthquake. Standing motor cars may rock slightly. Vibration like passing of truck. Duration estimated.

IV. During the day felt indoors by many, outdoors by few. At night some awakened. Dishes, windows, doors disturbed; walls made cracking sound. Sensation like heavy truck striking building. Standing motor cars rocked noticeably.

V. Felt by nearly everyone; many awakened. Some dishes, windows, and so forth broken; a few instances of cracked plaster; unstable objects overturned. Disturbance of trees, poles, and other tall objects sometimes noticed. Pendulum clocks map stop.

VI. Felt by all; many frightened and run outdoors. Some heavy furniture moved; a few instances of fallen plaster or damaged chimneys. Damage slight.

VII. Everybody runs outdoors. Damage *negligible* in buildings of good design and construction; *slight* to moderate in well-built ordinary structures; *considerable* in poorly built or badly designed structures; some chimneys broken. Noticed by persons driving motor cars.

VIII. Damage *slight* in specially designed structures; *considerable* in ordinary substantial buildings with partial collapse; *great* in poorly built structures. Panel walls thrown out of frame structures. Fall of chimneys, factory stacks, columns, monuments, walls. Heavy furniture overturned. Sand and mud ejected in small amounts. Changes in well water. Persons driving motor cars disturbed.

IX. Damage *considerable* in specially designed structures; well designed frame structures thrown out of plumb; *great* in substantial buildings, with partial collapse. Buildings shifted off foundations. Ground cracked conspicuously. Underground pipes broken.

X. Some well-built wooden structures destroyed; most masonry and frame structures destroyed with foundations; ground badly cracked. Rails bent. Landslides considerable from river banks and steep slopes. Shifted sand and mud. Water splashed (slopped) over banks.

XI. Few, if any (masonry), structures remain standing. Bridges destroyed. Broad fissures in ground. Underground pipe lines completely out of service. Earth slumps and land slips in soft ground. Rails bent greatly.

XII. Damage total. Waves seen on ground surfaces. Lines of sight and level distorted. Objects thrown upward into the air.

The assignment of the intensity of a given earthquake in a given locality is best based on observations made or col-

lected by a trained seismologist who visits the locality shortly after the shock. If the damage was great, the investigator may observe for himself. If it was slight, he must make many inquiries of persons who felt the shock. Here he may run into difficulties. In some regions such as parts of eastern Canada he may find the inhabitants proud of the shock and inclined to exaggerate the effects. In most parts of California he finds the local people heartily ashamed and inclined to conceal or even to deny the effects. Only by inquiring of a number of people may the general effects be best brought out. In the author's experience young men are the frankest, presumably because they own little property and are not in fear of their parents. It is best on making inquiries to appear important, to attempt to place the weight of some institution behind you, and not casually to inquire at the roadside as though curiosity were the motive.

In assigning intensity there are decisions to be made. For example, if only one vase on one mantel is discovered to have tumbled over, should the intensity be set at V (unstable objects overturned)? The author always rates the intensity by the maximum effect observed or reported unless very exhaustive information is available and it is known with considerable certainty that there were no other unreported phenomena of the same nature. Again, certain outdoor phenomena such as conspicuous faulting may occur within a few feet of a house where the chimney did not fall (the Bear Valley ranch house in 1906). These are usually isolated cases, however. The occurrence of slumps and landslides depends largely on the condition of the soil and may occur without an earthquake.

There is a practice, which the writer deplores, of lowering the observed intensity on the ground that it was due to poor geologic foundation (loose soil or fill) or to especially flimsy construction. Let the rated intensity be a true picture of the damage done! Point out, if necessary, why the intensity was so large locally, but do not hide the evidence in a

quest for an abstraction called the "true" intensity while depreciating the actual observation as "apparent" intensity!

Intensities, particularly in regions where the earthquake was slight, must usually be rated from written reports of local observers. In such cases much depends on the reporter. If he inquires among his acquaintances, he may give a fair value for the intensity. If he gives only his personal experience, his report may not be so valuable. The United States Coast and Geodetic Survey issues a post-card questionnaire which is circulated widely after earthquakes. On this the observer may check items regarding the intensity in his locality like those used in fixing the grade on the Modified Mercalli Scale. It is necessary that reporters be guided by such questionnaires. Many an individual who considers himself much interested in earthquakes and is willing to write a long letter reporting a shock actually devotes the whole report to his theory as to what caused it.

When the intensity in each locality has been rated, the values are usually plotted on a map. "Isoseismal" lines are then drawn so as to separate areas of one intensity from those of another (Figure 19). In general the resulting "contours" show a high at the center, with regions of lower intensity surrounding this area and more or less concentric with it. Some seismologists feel that unless the isoseismals are smooth curves concentric to a point, the whole process has been valueless. This feeling harks back to the days when locating the epicenter by this means was the sole approach to it. To the author the interesting feature of the isoseismal map is its irregularity. The irregularities point to regions of bad geologic foundations, perhaps to regions of particularly poor construction of buildings, but at any rate to localities in which earthquake hazard is greater. That is why isoseismal maps are drawn today. If the seismologist "doctors" his intensities in order to remove the effects mentioned above, the value of his work is greatly lessened, although he does get geometric isoseismals.

As stated above, the primary purpose of drawing isoseismal maps is no longer the location of epicenters, since these are now usually obtained by instrumental means. However, the maps serve as a check on the instrumental

Fig. 19. Isoseismal Map of the Earthquakes of October 2, 1934.

location. Speeds of waves in various regions are known only as general averages, which may very well vary locally. Assumptions of uniformity of speed in all directions and even errors in reading and interpretation of records are not impossible. Enough is not yet known about the earth's surficial structure for us to view with equanimity the loca-

tion of an instrumental epicenter in a region too far from the center of greatest intensity according to field evidence.

There are thus two kinds of epicenters, the field epicenter and the instrumental epicenter. The field epicenter is the region of greatest damage caused by the shock, or, in exceptional cases in which there is a surface fault break, it may be considered the trace of the fault. The field focus of the shock is that region, presumably under the epicenter, in which occurred the disturbances that sent out the earthquake waves. If a fault was the source, then it is the region near the surface of the fault. The instrumental focus (located from P waves) of an earthquake is the point in the earth's interior at which the disturbance started. If the earthquake is caused by faulting, it is the place where the fault started to break. From our knowledge of how bodies break and how paper and cloth tear, we conclude that it is highly improbable that a fault would start to break at several places at the same moment. A fault no doubt begins to break at one point and then tears along its length. Now the tearing can take place at a speed no greater than that of longitudinal elastic waves, for that is the greatest speed at which any mechanical disturbance can be propagated in an elastic medium. It is probable that faults tear at a considerably slower rate than that. Therefore, the first P wave to arrive at any station comes from that part of the fault that started to break first. One may quibble as to whether the focus is a "point." However, the first point on the fault to break offers the first increment of displacement to build up the first P wave. True, if only one "point" broke, that is, one molecule rubbed another, there would not be enough energy released to record across the world. Nevertheless, if this increment of energy is immediately reinforced by countless others, the net result will be a wave which may be registered on seismographs afar. The instrumental epicenter of an earthquake is the point on the surface of the earth immediately above the instrumental focus. (If the fault tears faster than S waves

travel, the first S at some stations may not have the same origin as P.)[3]

As stated above, it is and still should be somewhat disconcerting to a seismologist if his instrumental epicenter fails to lie within the field epicentral area. However, it is conceivable that a highly strained fault might be weaker (have less frictional resistance) in a region in which only a minor amount of strain was present; then it might break here and tear along to regions of high strain and therefore greater displacement. In such a case the instrumental epicenter might lie without the field epicenter.

A more complete description of an earthquake can be given by a scale which consists of several numbers. For instance, we may take four numbers: the first representing the intensity on some scale such as that given above, the second representing the greatest distance from the epicenter at which the shock was felt, the third representing the number of lives lost, and the fourth the property damage. Let us group as one, under grade 9, the last four grades, IX, X, XI, XII, of the Modified Mercalli scale. The first figure in our expanded scale will represent the maximum intensity as rated on this "modified" Modified Mercalli scale. We may then subdivide the other types of intensity measures also into nine groups, so that a maximum shock will be 9-9-9-9. We subdivide as follows, where r represents the maximum distance from the center in miles to which the shock was felt, N the number of people killed, and D the property damage in dollars:

Intensity	r	N	D
1..........	<10	<10	$<10^4$
2..........	10–50	10–50	10^4–10^5
3..........	50–100	50–100	10^5–10^6
4..........	100–200	100–500	10^6–10^7
5..........	200–350	500–1,000	10^7–10^8
6..........	350–500	1,000–5,000	10^8–5×10^8
7..........	500–750	5,000–20,000	5×10^8–10^9
8..........	750–1,000	20,000–100,000	10^9–2.5×10^9
9..........	>1,000	>100,000	$>2.5 \times 10^9$

Estimation of the damage in dollars is possible only for more recent shocks. When it is not possible to estimate one of the four numbers, the letter *u* (for unknown) may replace it. Fire damage is included with earthquake damage.

On this compound scale the intensities of a few well known shocks are:

California, April 18, 1906.........8-6-4-6.
Japan, September 1, 1923.........9-5-9-9.
Santa Barbara, June 29, 1925.....8-5-2-4.
Long Beach, March 10, 1933.......8-5-4-5.
Imperial Valley, May 18, 1940....8-6-1-4.

There is especial difficulty in fixing the first two numbers. To give the intensity on the Modified Mercalli scale or a similar scale is usually troublesome. Damage is rarely "considerable" in "specially designed" structures, and it is therefore rarely that one needs an intensity greater than 8, if damage to structures is the criterion. In the Japanese earthquake of 1923 the intensity in Tokyo should probably be rated no more than 8. The intensity 9 was given on damage in villages, and in these there were probably no "specially designed" structures. Again, to get the figure 6 for the second number of the rating for the 1906 earthquake the center was taken as the Golden Gate and the distance measured north. If this distance had been measured from the northern end of the fault, the figure would have been reduced to 5; similarly if the distance had been measured east from the fault.

For all that we have used four numbers, the great length of the region in which the 1906 earthquake was severe is still largely hidden.

Effects on earth's surface

As we have said, occasionally great earthquakes are accompanied by continuous fault breaks at the surface extending for some distance. In such cases the faulting

must penetrate to considerable depth. Far more often such fissuring and cracking as occurs on the surface is disjointed and discontinuous, apparently a failure of weak surface rock and soil to withstand the severe shaking or strain set up by large displacements of rock at depth. This type of fissuring is a *result* of the earthquake. It may occur in a linear band that follows the general trend of faulting in the region, in which case it seems definitely to point to more homogeneous faulting at depth.

Fig. 20. Earth Slump East of San Pablo, earthquake of 1906. (From the *Report of the State Earthquake Investigation Committee* published by the Carnegie Institution of Washington.)

Throughout the epicentral area of a large earthquake there are numerous disturbances of soil and rock which may be classified as follows:[4]

1. Slumps. In the rainy season the regolith or mantle of soil and loose rock on hillsides is often sufficiently wet to be unstable. A slump occurs when a portion of the wet regolith on the hillside slips down a little, rotating somewhat on a horizontal axis parallel to the hillside, the lower portion of the displaced mass rising relative to the upper

portion. A scarp is formed usually convex to the axis of the hill. Figure 20 shows a slump. Slumps frequently occur without earthquakes, but a large shock accelerates their formation and in a wet season their formation is a typical earthquake phenomenon.

Fig. 21. Earth Avalanche, Deer Creek, Santa Cruz Mountains, earthquake of 1906. (From the *Report of the State Earthquake Investigation Committee* published by the Carnegie Institution of Washington.)

2. Earth avalanches. Masses of dry earth and rock on steep hillsides are frequently loosed by a large earthquake. The masses may be very large, and they may slide for considerable distances. A classical example was the great earth avalanche in the Santa Cruz Mountains at the time of the California earthquake of 1906. Near Grizzly Peak an avalanche involving 25 acres flowed one half mile, with a change of elevation of 500 feet (Figure 21). It mowed down 10 acres of redwood trees, some 200 feet high, and covered them with debris.

3. Earth or mud flows. The severe shaking of underground water-bearing strata and displacements within them often cause water to be forced up through open fissures. Water so expelled may render the surface soil fluid, so that it flows as a liquid. Such flows occur at time of shock and do not recur. An example of such a flow is the one which occurred in Mount Olivet Cemetary, San Francisco, in 1906 (Figure 22). Some 90,000 cubic yards of sand and loam

Fig. 22. Earth Flow at Mount Olivet Cemetary, earthquake of 1906. (From the *Report of the State Earthquake Investigation Committee* published by the Carnegie Institution of Washington.)

flowed about 900 yards in 3 minutes. The mud attained a height of 13 feet and broke down trees. It left 3 feet of muddy sand on the bottom of the arroyo down which it flowed. The water that caused it gushed from an auxiliary fault fissure.

4. Earth lurches. Severe earthquake shaking of alluvial bottom lands frequently causes a lurching of the earth accompanied by the formation of cracks, usually parallel to the stream. The new position of the soil is not more stable than the old one. The inertia of the surface soil merely

prevents its partaking of the severe motion of the earth beneath it. Such phenomena were common in 1906 in the floodplains of the Eel, Russian, and Pajaro rivers (Figure 23).

Fig. 23. Earth lurch near Salinas River, earthquake of 1906. (From the *Report of the State Earthquake Investigation Committee* published by the Carnegie Institution of Washington.)

Effect of earthquakes on underground water

As was stated under the discussion of mud flows, the severe shaking of a great earthquake frequently causes shifts in water-bearing strata which result in changes in surface flow. *Earthquake fountains* which flow for a few minutes or hours sometimes result. In 1906 one such fountain in the Salinas Valley threw water 20 feet in the air although it flowed only a few minutes. It brought up sand from 80 feet deep. These fountains leave a crater of sand about them (Figure 24). Sometimes the sand is brought up by gases without much or any water. This phenomenon is called a *sand blow*. In the great Assam earthquake of 1897 sand from fountains and blows covered vast tracts of arable land and greatly hindered the farmers.

The flow of springs is greatly affected by a large earthquake. Some cease flowing or break out in new locations. Others experience an increase in flow. Frequently the water becomes muddy. Steamboat Springs (all boiling) near Reno in Nevada became muddy after the 1906 earthquake in California. Some hours or days after the shock

Fig. 24. Sand Crater Near Watsonville, earthquake of 1906. (From the *Report of the State Earthquake Investigation Committee* published by the Carnegie Institution of Washington.)

spring flows usually return to normal. In the Lompoc (California) earthquake of November 4, 1927, springs at the head of a small canyon near Lompoc were greatly reduced in flow, while new springs were formed near the foot of the canyon. After about six hours the new springs ceased to flow and the old springs resumed.

Maximum motion

The question as to what is the maximum amplitude of motion during an earthquake is difficult. In the first place there is the displacement along the fault which remains after the shock. As stated elsewhere, in the California

earthquake of 1906 there was a relative displacement of the two sides of the San Andreas Fault amounting to 21 feet at the head of Tomales Bay. On the Bear Valley dairy ranch the fault broke between the house and the front garden, with a relative displacement of 12 feet (see Figure 26). Yet the brick chimney on the house did not fall. When one speaks of maximum amplitude of earthquake motion, he means amplitude of shaking in localities where the ground returns to its original position after the vibrations have passed. He does not mean fault displacement or the lurching or sliding of ground which is poorly consolidated.

From observation of the shifting of monuments and the like after a great shock it may be difficult to distinguish between those displacements due to elastic vibration and those due to relatively small permanent shifts of the underground. Certainly the amplitude of seismic waves depends on the geologic foundation. It is greatest on made land and least on hard rock. After the great Assam earthquake of 1897 it was concluded from the displacement of two masonry tombs at Cherrapunji that the range of motion was about 14 inches[5]. Integration of the records of the United States Coast and Geodetic Survey's strong-motion seismograms written at El Centro during the Imperial Valley earthquake of May 18, 1940, indicates a maximum horizontal range of motion (double amplitude) of about 15 inches, accomplished in less than 3½ seconds. The maximum vertical range of motion was 4 inches, accomplished in 2½ seconds and occurring a little later than the maximum horizontal motion. The maximum acceleration exceeded 0.3 gravity on the north-south component, attained almost 0.3 gravity on the vertical component, exceeded 0.2 gravity on the east-west component. On the horizontal components this maximum acceleration accompanied periods of about 0.4 second, whereas on the vertical the period was about 0.2 second. After the great Tokyo earthquake of September 1, 1923, many low wooden structures at Otawara and Kodu had been displaced 25 to 50

inches. Imamura credits this to a succession of impulses rather than one earth displacement.

It is difficult to conceive of such large amplitudes occurring without complete destruction. Perhaps some of them were permanent displacements of alluvium rather than elastic waves. However, the existence of accelerations exceeding 0.3 gravity in such moderate earthquakes as the Long Beach (1933) shock and that of the Imperial Valley (1940) was almost unbelievable when first reported. These values are sound, however. The short periods at which they occur and the shortness of their duration remove them far from static forces of such magnitude. And we know that buildings carefully designed and constructed with earthquake hazard in mind have been able to weather these shocks.

It is interesting that on the Coast and Geodetic Survey's strong-motion seismograms of the three destructive earthquakes so far recorded (Long Beach, Helena (1935), Imperial Valley), the maximum horizontal acceleration has been around 0.3 gravity and has accompanied periods of around 0.3 to 0.4 seconds.

Effects of earthquakes on the sea

Sea quakes. Earthquake waves of the compression-rarefaction type may be refracted at the ocean bottom into the water. The resulting vibrations in the water are called *sea quakes.* They may be observed by mariners as a ruffling of the surface, a "short chop." The sensation on the ship is that it has run aground or hit a submerged log. In a large sea quake the masts shake and the whole ship trembles. Fish are frequently stunned or killed by sea quakes. Fishing grounds are sometimes ruined by the migration of fish from the epicentral region of a large quake. The focus of an earthquake which sets up sea quakes may be either under the ocean or under the adjoining land. As long as the compression-rarefaction waves incident on the ocean bottom are strong enough to set up sea quakes, the position of the focus is not important.

Seismic sea waves. The type of sea disturbance best known may be called a *seismic sea wave*, although the Japanese term *tunami* is frequently used. In popular parlance these waves are known as *tidal waves*. This latter is a misnomer, since they have nothing to do with the tides. They are gravity waves first set up by a large disturbance of the sea bottom—a vertical displacement of the bottom due to faulting or a submarine landslide. This results in the sudden formation of a mound or trough of water at the surface of the ocean above the disturbance. Gravity immediately acts to level the surface. The inertia of the water rushing toward a level carries it beyond equilibrium, and an oscillating wave is set up which travels out from the source with a theoretical speed of $v = \sqrt{gh}$, where h is the depth of the water and g the acceleration due to gravity. These waves lessen in amplitude as they spread out from the source. Their length at large distances is frequently measured in hundreds of miles, and their amplitudes in feet. They are therefore not noticed by ships at sea. On arrival at near-by coasts, however, their effect may be large. According to Imamura[7], tunamis are most destructive in very deep V-shaped bays with broad mouths, the depth increasing with distance from shore. As the wave advances up such a bay, the front steepens and the height increases until finally the wave breaks, flooding the low-lying ground in the vicinity. Arriving at distant coasts, the waves are observed only on tide gauges, on which they resemble tides of short period, that is, periods measured in a few tens of minutes.

The tunami of June 15, 1896, which flooded the Sanriku district in Japan resulted in the destruction of over 10,000 houses and loss of more than 27,000 lives. The center of the submarine earthquake which resulted in the wave was some 200 kilometers off shore, so that there was no earthquake damage in the region. The tunami did not arrive until an hour after the earthquake was felt. The wave attained a height of 30 meters at Ryôri. Six large waves

at intervals of from 7 to 34 minutes swept the town of Miyako.

The Sanriku tunami was recorded on the tide gauge at San Francisco some 4,780 miles away. It crossed the Pacific in 10 hours and 34 minutes with an average speed of about 450 miles per hour.

The tunami accompanying the Iquique (Chile) earthquake of 1877 recorded at Hakodate (Japan) as a rise and fall of the water with a period of about twenty minutes and a duration of several hours. The maximum amplitude was 8 feet. Hakodate is about 10,300 miles from Iquique.

Seismic seiches. Closed or nearly closed bodies of water such as lakes or land-locked bays have certain natural oscillations into which they may be set by varied causes. The arrival of seismic sea waves is one such cause. Although the tunami itself may force its own period on the bay, after the tunami's first force is spent the bay may continue to oscillate in its own free period for a long time. The Iquique tunami set San Francisco Bay into oscillation which continued for more than two days. According to Davison, the free-period lateral oscillation of the bay (that is, between West Berkeley and Sausalito), 34 to 41 minutes, was prominent. Shaking by the earthquake itself frequently sets bodies of water into seiche motion. The great Lisbon earthquake of November 1, 1755, set bodies of water into oscillation throughout Europe. The River Dal (in Sweden), over 1,800 miles from Lisbon, was severely disturbed by the earthquake.

Earthquake sounds

Very frequently accompanying earthquakes, often preceding them, and sometimes following them, there are audible noises of low pitch. Since the first preliminary seismic waves are longitudinal and since some portion of them must be refracted into the air, where they must have the nature of sound waves, it is not surprising that earthquakes should be heard if the frequency falls in the audible

range. All that is required is a sufficient intensity of the vibrations of the correct frequency. High frequency waves are quickly damped out; hence the relative infrequency of earthquake sounds and their low pitch when heard. On account of their low pitch, earthquake sounds are not heard by all, some ears being tuned to a higher pitch than others. They are usually heard, if at all, in the area around the epicenter, although in many cases Davison has found that the area of hearing and that of feeling are not concentric. The sounds are compared with distant thunder, roaring of wind, sounds of blasting, and so on. Davison has given a scale on which these different types of sound are classified.

Earthquake sounds must not be confused with the rattling and creaking of parts of houses or furniture which regularly accompany an earthquake or with the rustling of trees due to shaking. Any hard shock will obviously disturb many objects, with attendant noises, but the term "earthquake sound" is reserved for those low sounds which cannot be attributed to the effect of shaking on surface objects. Many times the sounds are heard without the shock being felt. Of the aftershocks on record of the Mino-Owari earthquake of 1891, over one tenth were merely "earth sounds." Again, there are regions on the earth's surface where earth sounds are common even though earthquakes are rare. In the district of East Haddam, Connecticut, earth sounds were common in the eighteenth century. They varied from sounds as of pistol shots to thunderous noises. Similar phenomena less surely correlated with a source in the earth have been called by various names in the localities where they are heard, for example, *mistpoiffeurs* in Belgium. They have all been classed under the name *brontides* by Allipi.

Many of the aftershocks of the Charleston (South Carolina) earthquake of August 31, 1886, were earth sounds. In a diary kept by Miss Ada M. Trotter,* who was in Summerville, a village near Charleston, in the springs

* This diary is now owned by Graham B. Moody of San Francisco.

of 1887 and 1888 and who kept a record of earthquakes felt in that period, occurs the following typical entry:

> *Mar. 13 to 25 (1888).* Scarcely a day or night has passed without several detonations and shakes. The former have been more or less loud, the latter sometimes sharp, but often very slight. I felt three during one hour of the 19th at midnight, as did others in the village. The effect on the house is found in the blinds, window catches and door locks which get out of gear and have to be hammered into place.
>
> *Mar. 25 to April 8.* Detonations more or less violent occur nearly every day—and shakes sometimes at the rate of four in one night. One on Wed. L, in the night one of the severest this winter. Property holders much discouraged.

One sympathizes with the property holders when the aftershocks continue to occur this often almost two years after the main shock.

Of eighty-one persons hearing earthquake sounds at the time of the California earthquake of 1906, forty reported hearing sounds before the shock and three hearing sounds after the shock.

There are frequent reports of animals being conscious of impending earthquakes. Perhaps animals are sensitive to vibrations of pitch below those men can hear and yet not of sufficient intensity to be felt by men. Horses in a stable in San Francisco at the time of the great earthquake reared and snorted just before the stableman felt the shock. A farmer near Santa Rosa, observing a commotion among his horses, called to his son asking the cause. Before the boy could answer he felt the shock himself.

Apparently earthquake sounds are more commonly heard in British earthquakes than in Japanese or California shocks, although they are not at all uncommon in the latter places. It may be, as Davison has suggested, that some races of people are able, on an average, to hear lower pitched sounds than others.

Earthquake lights

Occasionally during an earthquake shock or immediately before or after observers report luminous phenomena in the heavens. All types of lights are reported seen, although it is rarely that two observers see exactly the same. There are steady glows, red and blue and white; there are flashes, balls of fire, and streamers.

At the time of the earthquake off the coast of northern California in January, 1922, one observer reported a glow at sea which he at first took to be a ship on fire. At the time of the earthquake of October, 1926, centering in Monterey Bay, an observer reported a flash at sea which resembled "a transformer exploding." During the Humboldt County (California) earthquake of 1932 an observer reported, "several of my friends and I saw to the east what appeared to be bolts of lightning travel from the ground toward the sky. The night was clear." It has been customary to attempt to ascribe earthquake lights to secondary phenomena, since we know of no source of such lights in the original earthquake action. True, movement on a fault would generate considerable heat, and after the Sonora earthquake of 1897 trees overhanging the fault were scorched. Such would scarcely produce flashes in the sky, particularly over the ocean. In modern times the prevalence of electric power lines enables one to explain away many such observations as due to probable breaks in such lines; but many may not be so dismissed. Landslides in mountains may generate great heat by friction. In the Owens Valley earthquake of 1872 fires were started in the mountains by such sources. In some cases thunder storms may happen to accompany earthquakes, and then lightning may be called upon to explain flashes. Lights over the sea have been attributed to luminous marine organisms excited by the earthquake vibrations.

It is to be hoped that in these days of numerous camera enthusiasts we may be able to get some photographs of earthquake lights. A few such photographs would be

very helpful, for they would remove the possibility of doubt as to the psychological content of the reports.

Sieberg's remark that the chapter on earthquake lights is the darkest in seismology was as sound as it was clever.

Foreshocks and aftershocks

Some large earthquakes are preceded by a series of smaller shocks called *foreshocks.* All large earthquakes are followed by a series of smaller ones called *aftershocks.*

There is a certain amount of arbitrariness in the labeling of a shock as a foreshock. It must come from very closely the same source as the large earthquake and occur not very long before it. In a region of large shocks one can conceive the problem as to whether all small shocks might not be called either aftershocks or foreshocks, with the question arising as to whether or not a given small shock should be considered an aftershock of the last big earthquake or a foreshock of the next.

A seismologist who holds to the elastic rebound theory of cause of earthquakes, that is, a slow accumulation of strain in the earth's crust with final release in rupture, may fear any series of small earthquakes as possible foreshocks indicating a condition of great strain and a consequent preliminary yielding of weaker parts of rock just before the great break. Or he may comfort himself that the small shocks are relieving the strain as it accumulates and rendering the locality safe from large earthquakes.

The Lompoc (California) earthquake of November 4, 1927, occurred at 5h 51m A.M. Two foreshocks were recorded at the Pasadena seismographic station (about 150 miles from the epicenter), at 3h 10m A.M. and 4h 20m A.M. Other shocks at 1h 10m A.M. and 2h 30m A.M. were reported felt in the epicentral region. They were slight, awakening only a few. On the other hand there are no authenticated reports of foreshocks preceding the great California earthquake of 1906. None is listed in the *Report of the State Earthquake Investigation Commission.*

At this late date one may find people living in San Francisco at the time who remember that there were foreshocks and others who remember that there were none.

An exact record of the number of aftershocks of a large earthquake is almost impossible to obtain, since there are so many of them and they vary so much in intensity and overlap each other. The writer spent the night in Lompoc on November 5, 1927, and found it impossible to record the exact number of shocks. They occurred very frequently, some being just slight quivers and one often merging into the next.

The record of the number of aftershocks of the 1906 earthquake includes about ninety shocks before noon. (The main shock was at 5h 12m A.M.) These were reported from various points along the fault, and some may be duplications. However, many probably went unrecorded.

Following a large earthquake the aftershocks often continue for years, that is to say, the region experiences an unusual number of shocks which die off with time, and it is not for some years that shocks become as infrequent as they were in the years preceding the great earthquake.

After the Mino-Owari earthquake of October 28, 1891, Omori made a study of the aftershocks recorded on seismographs at Gihu near the epicenter. Three hundred and eighteen shocks were recorded in the first day following the earthquake and 1,746 in the first month. Omori found that the decline in frequency of the aftershocks followed fairly closely an hyperbolic law.

The incessant recurrence of aftershocks after a great earthquake is most unnerving to the populace. Stories are spread about that the worst is yet to come, and panic is easily engendered at such a time. Although a few exceptions may be cited, it generally holds that a destructive earthquake is not followed closely by another of the same magnitude but by a series declining in intensity and frequency.

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CHAPTER VI

Distribution of Earthquakes

North America

The most seismic part of North America is along its Pacific Coast. From the Aleutian Islands south to Central America there are many earthquakes. The Aleutian Islands are very seismic, many world-shaking earthquakes centering in them. The Alaskan and Canadian Pacific coastal regions are also quite seismic, although northern Alaska and the interior of Canada are quiet. The region near the Queen Charlotte Islands has been the seat of a number of large earthquakes. During historic time there has been a lack of very heavy shocks on land and very few submarine shocks in the region between Vancouver Island and northern California. Off the coast of northern California there are many and heavy submarine shocks (19 large ones between 1865 and 1937). South of Point Arena there are no submarine centers at any distance off the coast until Point Arguello is reached, although there have been a few epicenters off the Golden Gate where the San Andreas Fault skirts the coast, in Monterey Bay, and close to the shore near San Simeon. In this region the main centers lie on shore in the province of the Coast Ranges and in the Owens Valley region. There have been a number of fairly heavy shocks westerly from Point Arguello. Large earthquakes do not center at any great distance off the coast of southern California. It is thus only at those places off the coast of California where the trend of the San Andreas Fault heads to sea that there are large submarine shocks, and these persist only to distances of about 100 miles from the coast. Beyond this the sea bottom is quiet until the seismic region about the Hawaiian Islands is reached.

The states of Montana, Nevada, and Utah are the most seismic of the mountain states east of California. A number of moderately large shocks occur in them.

In central Mexico the seismic zone broadens to include the width of the country, all of Central America, and much of the West Indies. Submarine earthquakes are not uncommon off the coast of southern Mexico.

As for the United States and Canada east of the Rocky Mountains, earthquakes are small and infrequent except for a few centers. The region in the neighborhood of the Gulf of St. Lawrence has from time to time been the seat of very violent earthquakes. In fact the most violent earthquake on record in North America, if early reports may be trusted, centered in or near the valley of the St. Maurice River in 1663. The Grand Banks earthquake of 1929 lies in this general territory.

A series of earthquakes in the fall and winter of 1811–12 centering about New Madrid, Missouri, makes it necessary to include southern Missouri as one of the great seismic areas of North America. These shocks were the most intense in the history of the United States. Small earthquakes still occur in the region. Likewise the great earthquake of 1886 makes it necessary to include the region about Charleston, South Carolina, as a major seismic district.

Submarine earthquakes off the Pacific Coast of North America have not resulted in the great sea waves which are so disastrous in South America and Japan.

South America

The seismic areas of South America include its entire Pacific coast and its Caribbean coast. The eastern part of the continent is quite free from shocks. Brazil was cited by Montessus de Ballore as an example of an aseismic country. All along the Pacific coast there have been great earthquakes. Chile has been the seat of many of them, as has Peru. Many of these shocks have been particularly

disastrous in that they have resulted in great sea waves. Although in a number of the earlier shocks elevations of large portions of the coast several feet have been reported, the reality of such elevations has been questioned. Lack of careful surveys and dependence on crude observations generally lead to the discredit of the conclusions when such significant ones are made.

A moderate seismic zone extends westward from the coast of southern Chile to include Easter Island.

Central America and the West Indies

With the exception of the Bahama Islands, the entire West Indies are seismic, although the southern part of Cuba is the seat of the large shocks. Central America lies in the heavy earthquake belt.

Europe

In Europe it is the territory adjacent to the Mediterranean that is seismic: the peninsulas of Spain, Italy, and Greece, and the Alpine and Carpathian regions to the north. There are a few small earthquakes in northern France, Great Britain, the Low Countries, and western Germany and also in southern Russia.

Asia and Australasia

The Asian seismic regions follow in general the Caucasus-Himalayan trend, including southern China. Another broad belt runs from the Pamirs to the region of Lake Baikal. Northwestern China also experiences great shocks. Southern India, most of the Arabian peninsula, and the central part of the Malayan peninsula are not seismic. The East Indies, the Philippines, the Japanese Islands, and Kamchatka are regions of strong earthquakes. They are in the belt connected by the Aleutian Island region to the seismic region of the west coast of the Americas. This circum-Pacific belt extends south by the Solomon Islands to include New Zealand. Australia has very few earthquakes.

Africa

The Barbary states have a fair degree of seismicity, the northern part being quite seismic. Northern Egypt and the Red Sea region are subject to some shocks, and this region of mild seismicity extends down the Rift Valley to 10° or 15° south latitude. The greater portion of Africa is, however, not seismic.

The Atlantic

Down the middle of the Atlantic Ocean there is a seismic region with occasional large shocks in Iceland, the Azores, St. Paul Rocks, the Sandwich Islands, and vicinities.

The Pacific and Indian Oceans

A number of large earthquakes occur in the region closing the circum-Pacific great circle zone in the south Pacific, but not so many as elsewhere in the zone. A zone extending westward from Chile as far as Easter Island is more seismic than the region between Easter Island and the New Zealand region. There are also a number of epicenters in a broad zone extending westward from Colombia through the Galapagos Islands and beyond, where the zone turns southwest to Easter Island. Occasionally a large shock is recorded elsewhere in the Pacific Ocean, but they are not common. In the vicinity of the Hawaiian Islands there are many small shocks and an occasional large one. The Indian Ocean is the seat of a fair number of shocks. In the south Indian Ocean there is an important center southeast of Madagascar and another between it and Australia.

General

The great seismic regions of the world lie mainly in two belts which follow great circles around the earth, the one surrounding the Pacific and the other passing through the East Indies, the Himalayas, the Caucasus, the Alps, and

the West Indies. According to data compiled by Montessus de Ballore some years ago, a little over half the earthquakes on record occurred on the second belt (some 54 per cent), while about 40 per cent occurred on the circum-Pacific belt. According to Gutenberg and Richter, about 80 per cent of the seismic energy of the earth is released in the circum-Pacific belt[3]. These zones are not seismic throughout, but the majority of earthquakes occur in them. Montessus de Ballore has pointed out that the greater the gradient of the earth's crust (the greater the slope), the greater the number of earthquakes which occur there. High mountains near deep seas mean earthquakes. This is a general rule, but of course large earthquakes have occurred in regions of relatively low gradient, for example, the great Charleston (South Carolina) earthquake of 1886.

Frequency of earthquakes

Many efforts have been made in the past to find in the frequency of occurrence of earthquakes some law which might lead to the prediction of future shocks. None of these has been successful as to prediction. A complete account of Charles Davison's work in applying the methods of harmonic analysis to the frequency of earthquakes in time is given in his recent book, *Studies on the Periodicities of Earthquakes*[1]. He examines earthquake lists for the following periods: annual, diurnal, eleven year, nineteen year, forty-two minute, lunar. The difficulty lies in interpreting the significance of the amplitudes in the harmonic analysis, since random numbers may be analyzed to show some periodicity.

Davison's analysis of earthquakes of the whole world based on the catalogs of instrumentally located epicenters from 1918 to 1930 shows a very strong tendency for the maximum number of shocks to occur in summer, specifically in July. Rodés[2] has shown a remarkable correlation between the occurrence of earthquakes as recorded on seismographs at Tortosa and Greenwich time. He con-

cludes, "The maximum frequency of earthquakes occurs when the regions of maximum seismic activity (from the Andes to the Aleutian Islands) pass in front of the sun carried by the rotation of the planet." The maximum occurs at about 21 hours Greenwich time. None of the tendencies is so strong, however, as to lead to prediction; that is, there are many shocks at times of minimum. At present it is not with prediction in mind that efforts to find periodicities are made, rather with the hope of finding some correlation which will give a hint as to the underlying cause of earthquakes.

Seismicity

"Seismicity" is a word which has no general quantitative definition. The seismicity of a region of many large shocks is, of course, greater than that of few small ones. Montessus de Ballore divided regions into three types: seismic (as Japan), peneseismic (as Switzerland), aseismic (as Brazil). A thorough rating of seismicity should take account of the area shaken, the intensity of shaking, and the frequency with which earthquakes recur. There is a certain degree of proportionality among these; a region of very frequent shocks is usually one in which there are great earthquakes affecting large areas. The accompanying map (Figure 25) illustrates the seismicities of various groups of counties in California. It is based on records of earthquakes from 1850 to 1937. The seismicity was rated as follows: All earthquakes on record which were not strong enough to disturb movable objects were weighted 1, all shocks stronger than this but which did not damage masonry were weighted 5, those strong enough to damage masonry were weighted 25. The few earthquakes which produced definite faulting at the surface were weighted 125. For each group of counties the total weights of all shocks centering within them were computed and the sum divided by the area of the region. The San Francisco Bay counties were given the arbitrary seismicity number of 10, and the

other groups of counties values in proportion to 10 as the sums mentioned above were to the sum for the San Francisco region. The three hyphenated numbers appearing above the seismicity number in each subdivision on the map

Fig. 25. Seismicity Map of California.

give successively the number of shocks (1) not strong enough to disturb movable objects, (2) strong enough to disturb movable objects but not strong enough to damage masonry, (3) strong enough to damage masonry. The

groups of counties are marked by capital letters. The following table gives the names of the counties in the groups so designated. Northern San Bernardino County was excluded from the grouping because of lack of reports from that region.

Group	*Counties*	*Seismicity*
A.........	Alameda, Contra Costa, San Francisco, San Mateo, Santa Clara	10.
B.........	Del Norte, Humboldt	4.54
C.........	Orange, Los Angeles	3.61
D.........	Imperial	3.57
E.........	Monterey, San Benito, Santa Cruz	3.05
F.........	Santa Barbara	2.94
G.........	Marin, Napa, Solano, Sonoma	2.61
H.........	Ventura	1.84
I.........	San Luis Obispo	1.69
J.........	San Diego	1.64
K.........	Lake, Mendocino	1.31
L.........	Riverside	0.64
M.........	El Dorado, Nevada, Placer, Plumas, Sierra	0.64
N.........	Inyo, Tulare	0.59
O.........	Kern	0.56
P.........	San Bernardino (lower half)	0.52
Q.........	Lassen, Shasta, Trinity	0.38
R.........	Alpine, Amador, Calaveras, Mono, Tuolumne	0.17
S.........	Butte, Colusa, Glenn, Sutter, Tehama, Yolo, Yuba	0.15
T.........	Fresno, Kings, Madera, Mariposa	0.14
U.........	Merced, Sacramento, San Joaquin, Stanislaus	0.12
V.........	Modoc, Siskiyou	0.04

In this evaluation of seismicity earthquakes were assigned only to counties in which they centered. (The 1906 earthquake was assigned equally to the four groups of counties in which the fault break occurred.) Thus "fringing" effects of large shocks centering in another group of counties are not counted. These are not usually great. However, the excessive severity of the 1906 earthquake in Los Banos is lost to Merced County by this method. In studying the map it will be well to remember that a high seismicity in one group of counties is likely to carry over the border to a neighboring group in a way not indicated by the low seismicity number of that second group.

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CHAPTER VII
Great Earthquakes

North America

The St. Maurice earthquake of February 5, 1663. This very great earthquake centered in the lower valley of the St. Maurice River, a tributary of the St. Lawrence, about halfway between Montreal and Quebec. According to the reports in *Jesuit Relations* and letters of Marie de l'Incarnation, founder of the Ursulines of Quebec, a very great waterfall on the St. Maurice was leveled at the time of the shock. "One sees today fields of more than a thousand arpents (an arpent was 1 to $1\frac{1}{2}$ acres) all leveled as if they had been recently tilled in many places where before there was nothing but forests"[1].

The New Madrid earthquakes of 1811–12. On December 16, 1811, began a series of earthquakes which shook the New Madrid region of southern Missouri for a year. Three of these shocks were major earthquakes. Such a sequence is unusual, the normal order being one major shock (perhaps preceded by a few small foreshocks) followed by a long series of aftershocks of lessening intensity. The first big earthquake came at 2 A.M. on December 16, 1811, with 27 aftershocks before morning and of course the regular series of aftershocks continuing. On January 23, 1812, occurred another major earthquake, and on February 7, 1812, came "the hard shock." Between December 16 and March 16, 1,874, of these shocks, of which 8 were severe, were felt in Louisville 200 miles away.

The epicentral tract was a strip of 30,000 to 50,000 square miles bordering the Mississippi River from New Madrid south. It included part of eastern Tennessee and northwest Arkansas. The very conspicuous feature was the

formation of the "Sunken Country." An area about 240 kilometers long by 60 kilometers broad sank 1 to 3 meters. River water rushed into this region, forming new lakes, swamps, and bayous. Reelfoot Lake in Tennessee (8 to 10 miles long and 2 to 3 miles broad) was formed at this time. Waves in the soil many feet high broke, leaving parallel fissures. Sand was ejected from fissures and craters; sulfurous vapors arose; dust filled the air, darkening the days. There was great destruction to timber; many trees were broken, many drowned, and many died as a result of damage to the roots. Low domes up to 20 feet in height, with corresponding depressions, resulted from the warping of the soil.

Chimneys were thrown down in Cincinnati 400 miles from the epicentral tract. In Washington, D. C., 800 miles away, doors and windows rattled. It is said that a church bell was rung by the shaking in Boston 1,100 miles away. The earthquake was felt as far away as the headwaters of the Missouri and the Arkansas Rivers, on the Gulf of Mexico, and in Canada. It is definitely the greatest earthquake on record in the United States.

Indian legends record an earlier earthquake in the same region before the white men came. Fuller states that he observed old fault scarps with trees 200 years old growing on them[2].

The Charleston earthquake of 1886. On August 31, 1886, at 9h 51m P.M. the city of Charleston, South Carolina, was shaken by a violent earthquake which killed 27 people out of a population of some 50,000. The number dying of exposure brings the figure up to near 100. The earthquake did damage amounting to $5,000,000. Few buildings were demolished. The main damage was to new buildings on filled ground.

The Charleston region had been quite free of earthquakes from 1680 to 1886. A few foreshocks occurred in the week previous to August 31, but residents were so unaccustomed to earthquakes as to doubt what it was which they felt.

An awful roar accompanied the main shock, which lasted 70 seconds. Many acres of land in the epicentral region were covered with sand from sand blows and earthquake fountains. The area in which these phenomena occurred was about 600 square miles. In this region soil was disturbed and railroad tracks were warped and twisted.

The earthquake was felt as far away as Boston, 800 miles; the upper Mississippi, 950 miles; Cuba, 700 miles; Bermuda, 950 miles. As far as area felt was concerned, this earthquake seems to have been of the same magnitude as the New Madrid earthquake. But the latter was much more severe in the epicentral tract. Whereas the New Madrid shock sent citizens of Charleston scurrying from their beds, the Charleston earthquake did not frighten the residents of New Madrid. Aftershocks including earth noises were very common for years. An extract from a diary kept in Summerville 22 miles northwest of Charleston was cited on page 75. It indicates the great frequency of aftershocks even after almost two years had elapsed.

The Charleston earthquake was unusual in several respects. It occurred in a region previously free of shocks. Although the area over which it was perceptible was tremendous (probably about 3,000,000 square miles), the size of the epicentral area was not great and the phenomena there were not extreme. Comparing it with the California earthquake of 1906 with a much larger (longer) epicentral area and comparatively small area of perceptibility, one is puzzled as to the nature of the source of the Charleston shock.

The California earthquake of 1906. This earthquake was accompanied by (no doubt caused by) the most magnificent surface fault break on record. On the morning of April 18, 1906 at 5h 12m A.M. the San Andreas Fault broke for a distance of 270 miles, from Upper Mattole in Humboldt County to San Juan in San Benito County. Part of its path was at sea, and the total length given is obtained by connecting the break heading to sea at

Point Arena with the break entering land again at Shelter Cove. (See Figure 11.) The displacement was almost wholly horizontal and attained its maximum at the head of Tomales Bay, where it displaced a road 21 feet, the westerly side moving northerly relative to the easterly side (Figure 12a). The displacement died away to nothing toward either end of the fault. It was toward the northern end of the fault only that there was any indication of a vertical displacement, and at no place did this exceed two or three feet. No appreciable sea wave was set up by the earthquake even though considerable parts of the fault trace were at sea. This absence is consistent with the smallness or lack of a vertical component of displacement and is common to earthquakes off the coast of northern California.

The effect of the great length of fault was a long and narrow epicentral zone and destruction in widely separated cities. The zone in which the earthquake was strong enough to throw down chimneys was about 350 miles long and 70 miles wide, extending from Humboldt Bay to King City. The earthquake was felt as far north as Coos Bay, Oregon, and as far south as Los Angeles, a total distance of 730 miles. It was felt as far east as Winnemucca, Nevada, about 300 miles inland. The total area shaken sensibly (including ocean bottom) was then about 370,000 square miles, a relatively small area when compared with that of the New Madrid or Charleston shocks.

The strong shaking lasted about a minute. The major losses occurred in the city of San Francisco, the metropolis of the region. Here a disastrous fire followed the earthquake and served to mask the actual destruction caused by the earthquake. Fire so frequently follows large shocks that it may well be considered an integral part of the earthquake. True, it is a part that may be eliminated by correct fire protection, but so may most of the direct earthquake damage be eliminated by proper design and construction. The number of lives lost owing to the earthquake, including the fires, has been estimated at 700 for the whole shaken

area. Freeman[3] estimates the total loss of property caused by the earthquake as $20,000,000 in San Francisco and $4,000,000 outside it. The fire damage in San Francisco he estimates as $400,000,000. Very great damage was suffered by the town of Santa Rosa about 20 miles from the fault. The damage seems out of proportion to this distance even when the geological foundation of the city on an alluvial fan is taken into account. Fire also followed the quake here, and the loss of life was at least 61 out of a population of 6,700. Also in San Jose, situated on alluvial bottom land, the damage was great.

The damage caused by this earthquake brought out very clearly the effect of geologic foundation. In San Francisco the damage to buildings on rocky hill slopes was minor. In little valleys between the spurs of the hills where the soil mantle was thin the damage was somewhat greater. On the sand hills damage was considerable, and on filled ground it was great. Only well built buildings can endure a severe shock on a foundation of loose soil.

Europe

The Lisbon earthquake of 1775. On November 1, 1755, Portugal, Spain, and northwestern Africa were visited by violent earthquakes which have become famous historically, partly because of the great violence of the principal shock, particularly in Lisbon, and partly because it centered in Europe. There were three great earthquakes in Lisbon on the morning of November 1, at about 9h 40m, 10h, and noon. The first was the greatest. As it was All Saints' Day, thousands of the inhabitants were congregated in the churches of Lisbon. Most of the churches were destroyed by the shock and the people in them killed. The number of casualties in Lisbon alone has been estimated at from 30 to 70 thousand out of a population of some 235,000. In Faro some 3,000 were killed, and in towns in northern Africa such as Fez and Mequinez in Morocco many thousands more were destroyed. Within 6 minutes in Lisbon

all large public buildings were ruined, and 12 to 17 thousand houses out of 20 thousand were rendered uninhabitable. The great fires which followed the shocks completed the destruction. The second great shock resulted in the engulfment of a new quay on which many people had taken refuge.

The main shock was of great length, 6 to 7 minutes. The area over which it was destructive was very great. Lisbon was near the northern end of this area, which extended as far south as Fez and Mequinez in Morocco, some 400 miles from Lisbon. Destruction occurred also in Algiers about 700 miles east of Mequinez and Fez. There was some damage to buildings in Seville, Cordova, and Granada in Spain. Davison feels that the widely separated centers of great intensity indicate more than one source.

The total area over which the Lisbon earthquake was felt is somewhat obscured by reports of violent shocks which destroyed buildings more than 1,000 miles from Lisbon (Caen and Ponza) which can scarcely be credited to the same source as the Lisbon shock. Again, as Davison points out, the oscillation of chandeliers and of bodies of water at places at great distances does not necessarily indicate that the earthquake was felt by people in these localities. Davison considers it fairly certain that the earthquake was felt over an area of between 1¼ and 1½ million square miles. Much greater estimates have been made by other writers. The shock was violent on ships at sea. As far away as Rendsburg in Germany, 1,390 miles from Lisbon, large chandeliers in a church were set swinging. Most remarkable was the disturbance of bodies of water at great distances from Lisbon. The medicinal springs at Toeplitz in Bohemia, 1,380 miles from Lisbon, were muddied by the shock. Throughout France, Holland, Switzerland, and Great Britain ponds, lakes, and rivers were rocked. Loch Lomond, 1,220 miles from Lisbon, was set into seiche movement for 1½ hours. At Lubeck in Germany, 1,400 miles from Lisbon, the river Trave rose suddenly four or five feet.

A large tunami resulted from the main shock. At Lisbon the sea first retreated. Then a great wave, estimated variously as 16 to 50 feet in height, rushed in and invaded the shore for as much as a half mile. Bridges and walls were wrecked by it. Many people were washed away, and ships were torn from their moorings and swept to sea. The first wave came in about 10 A.M. Three more of height about sixteen feet occurred by 2 P.M., when the oscillation ceased. At Cadiz there were five or six waves. The first is reported to have been 60 feet in height. It tore away the city wall, carrying loads of 10 tons for 50 yards. At Kinsale in Ireland the wave rushed into the harbor, raising the level 5½ feet. At Antigua in the West Indies, 3,540 miles across the Atlantic, there were several waves of height 12 feet. The speed of the wave across the Atlantic was about 600 feet per second[4].

About 250 aftershocks were noted within the first six months.

Asia

The Indian earthquake of 1897. At about 5h 15m P.M. on June 12, 1897, an earthquake of major intensity, perhaps the most severe on record, shook southern Asia. It centered in the province of Assam at the head of the Bay of Bengal. The region of greatest intensity embraced about 30,000 square miles. Within this area there was practically total destruction of all brick or stone buildings. Within this area of maximum destruction Oldham[5] outlined a hat-shaped area of about 11,000 square miles in which there were dislocations of the ground, changes in position indicated by surveys, and vast numbers of aftershocks. Most of the inhabitants of the epicentral region were out of doors at 5h 15m P.M., so the death list was comparatively small, about 1,500. Many of those killed were crushed by landslips.

Serious damage to buildings occurred in an area of about 150,000 square miles. The total area in which the earth-

quake was felt is estimated by Davison as 1,750,000 square miles. However, there were reports of the shock being felt at isolated points at vast distances, for example at Livorno and Spinea in Italy (some 1,200 miles away). It is possible that local shocks were felt there at the time of the Indian earthquake.

The most violent part of the shock was probably a little less than one minute, with a total duration of perhaps three minutes. People were thrown to the ground and mauled by the shock; some were thus injured. The vertical acceleration was so great that boulders were thrown vertically upward, leaving the cavities in the earth in which they had lain with sides almost unbroken. Such action was observed at points 100 miles apart. Near Nongstoin a splinter of granite 3 feet by 1 foot was thrown 8½ feet. Many houses were sunk into soft soil until only their roofs were visible.

Visible waves on the ground were noted widely. Oldham has estimated the size of these waves as about 30 feet long by 1 foot high. A few short faults broke in the epicentral area. The most notable was the Chedrang Fault near the town of Jhira. This fault trends nearly north and south. It broke for about 12 miles. The maximum displacement was 35 feet vertical, the east side being elevated with respect to the west. No definite horizontal displacement was present. Another small fault a few miles to the south broke for 2½ miles. It trended northwest. It showed a maximum vertical displacement of 10 feet, the southwest side moving up relatively.

Some 50 miles east of the Chedrang Fault near the town of Bordwar a fracture of the earth and rock broke open for 7 miles but showed no displacement. Landsliding and destruction to timber near this fracture were great. The fissure was only a few inches wide and trended nearly east-west.

Other smaller evidences of faulting were observed in the region. But the surface faulting observed seems insufficient to have caused so large a shock. Oldham postulated

the breaking of a low-angle thrust fault as the cause of the earthquake, with the surface faulting as secondary.

Landslides were very common and disastrous in the epicentral region. On the southern slopes of the Garo and Khasi Hills they were especially prevalent. The hills were stripped bare of their forest covering by slides for some twenty miles. Fissures in the alluvium were common over an area 400 miles by 350 miles. Level rice fields were left in low swells after the passage of the earthquake waves. Telegraph poles in the alluvium were displaced as much as 15 feet to one side of the general line by lurching of loose soil. Earthquake fountains ejected sand to such an extent as to hinder farmers in later cultivation.

There were many aftershocks in various parts of the disturbed area. Up to the end of 1898, there are on record 5,523 aftershocks, coming from many different origins within the region disturbed by the great shock.

The Mino-Owari earthquake of 1891. The Mino-Owari earthquake is conspicuous, along with the California earthquake of 1906, in that it was accompanied by long and continuous enough surface faulting as to assure faulting as a sufficient cause for the earthquake. In general we have to infer faulting at depth as a cause. For this earthquake the major surface faulting confirms the soundness of our reasoning.

It was at 6h 37m A.M. on October 28, 1891, that this earthquake occurred. It centered in the provinces of Mino and Owari near the center of the main island of Japan. The main fault broke for a distance of at least forty miles and probably sixty or seventy miles. It was traced by Koto for 40 miles from the village of Katabira to the mountain Haku-san. Reports indicate that it extended through the mountainous region as far as Fukui. The trace was somewhat curved but trended generally northwest. Along the fault the eastern (Pacific side) moved northerly relative to the western side. Generally, Japanese faulting indicates a motion of the Pacific side northerly, and this is what is

observed in California, showing a remarkable uniformity of earth forces on the two sides of the Pacific Ocean. The maximum horizontal displacement of the Mino-Owari Fault occurred in the Neo Valley and amounted to 13 feet. The maximum vertical displacement was nearly twenty feet at Midori. At some points the west side was uplifted, and at others it was downthrown relative to the eastern side. In the most severely shaken district comprising something over 4,000 square miles the destruction to the works of man was almost total. Homes were thrown flat; embankments and railroads were severely damaged; over 10,000 bridges required rebuilding or repairs. More than 7,000 casualties resulted, and some 17,000 people were injured. Almost 200,000 houses were entirely destroyed. Omori estimated the total land area shaken as 96,500 square miles, corresponding to about 330,000 square miles as total area of shaking. This is not large considering the extreme intensity near the center and comparing this shock with others such as the Charleston earthquake.

There were great landslides in the epicentral area; many fissures appeared in the plains; and other typical earthquake phenomena occurred. Compression of the whole Neo Valley was indicated, according to Milne. It was on the basis of observation of the numerous aftershocks of this earthquake that Omori developed his hyperbolic law of the dying off of frequency of aftershocks. Three thousand three hundred and sixty-five aftershocks were recorded at Gifu near the fault during the first two years after the earthquake.

The Kwanto earthquake of 1923. The Kwanto earthquake was one of the most disastrous of modern times. It is named after the district at the head of Sagami Bay in which the city of Tokyo stands (see Figure 26). The earthquake occurred on September 1, 1923, at 11h 59m A.M., a time at which a great many inhabitants of Tokyo were in the downtown area. Although the shock centered in Sagami Bay, it was in Tokyo, some 57 miles, and Yokohama, some 40 miles away, that the effects were most spectacular. In Tokyo the most violent part of the motion

lasted less than 30 seconds, followed by slower undulations which lasted several minutes. Great fires broke out immediately, and much of the city was soon in flames. Before night more than 1 million out of 2½ million inhabitants were homeless. The great damage due to the shaking was in the

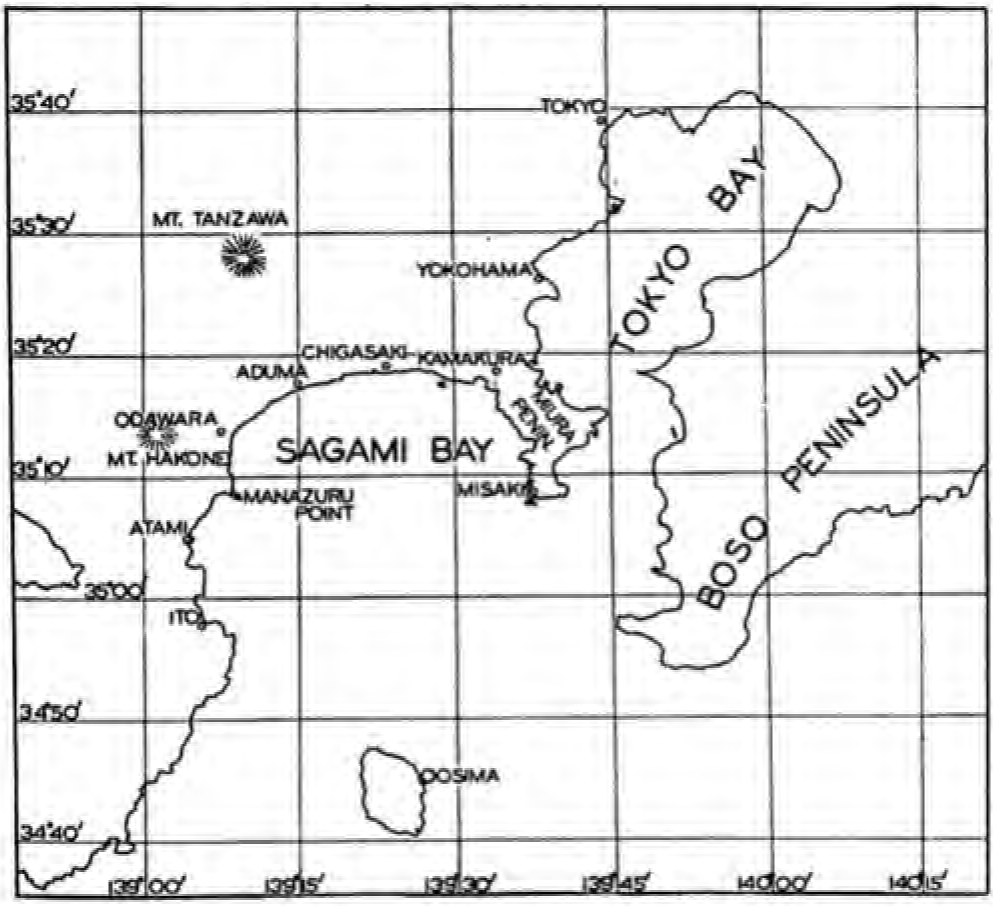

Fig. 26. Map of the Tokyo Bay Area.

downtown section, which was built on soft soil and fill. The upper town on firmer soil experienced much less shaking. The earthquake badly damaged or destroyed about 54 per cent of the brick buildings and 10 per cent of the reinforced concrete buildings. There were 16 large steel framed buildings, of which 6 were unharmed, the others being damaged. The death roll in Tokyo was some 68,000; many of these were due to fire. In the yard of the Military Clothing Depot alone 40,000 people were burned. The total dead in the whole epicentral district numbered almost 100,000, with an equal number injured and 43,000 missing.

Yokohama also was burned, the death roll there being about 23,000.

The total property loss was more than $2\frac{1}{2}$ billion dollars. Many smaller towns nearer the center in Sagami Bay were practically erased from the landscape, for example, Odawara, a place of some 5,500 houses. Fire again played its part. At Kamakura 84 per cent of the houses collapsed. Throughout the region it was the towns on loose alluvium which experienced the terrific destruction. Towns on good rock foundation were very much less damaged.

There was no great surface fault observed, although vast changes in depth in Sagami Bay are interpreted by some as indicating faulting in the bay bottom. Fifteen short faults were observed on the Boso peninsula and the west side of Sagami and Tokyo bays. The longest, of 13 miles, ran east-west near the southern end of the Boso peninsula. The greatest displacement observed was a 9-mile fault on the western side of Tokyo Bay. It was 6 feet 7 inches and was vertical.

Leveling operations were undertaken after the earthquake and carried out between September 1923 and March 1927. These were compared with the results of earlier leveling done in 1884 to 1899. The Standard Datum was in Tokyo, and it was taken as 86 millimeters lower than at the time of the earlier observations. This depression of the Standard Datum was concluded from comparisons with mean sea level and from the results of precise leveling connecting the Standard Datum with bench marks at points outside the region disturbed by the earthquake. The results of the leveling indicated that the tip of the Boso peninsula had risen 1.9 meters between the surveys; the lower end of Miura peninsula had risen at points as much as 1.4 meters. At Aduma on the north coast of Sagami Bay the rise was 1.9 meters. There were also some areas of depression. Fifteen miles northeast of Aduma there was a region of depression as much as 1.6 meters near its center. Estimates of changes of level by eyewitnesses during and shortly after the earthquake are more startling. Suda reports that at Misaki on the Miura peninsula the ground rose 24 feet 11 inches on

September 1, only to sink gradually until, on September 26, it was only 4 feet 7 inches above its former level.

Old triangulation stations occupied in 1884 to 1899 were reoccupied between May 1924 and October 1925. Computations of horizontal shifts of these points were based on (1) holding station Teruisiyame fixed and (2) holding station Kokusidake fixed. The displacements differed somewhat under the two assumptions but showed some tendency (with exceptions) to indicate a clockwise rotation about a center in Sagami Bay. One of the largest indicated displacements was that of the island of Oosima, which had shifted over three and one-half meters northerly. How much of these shifts in elevation and position occurred at the time of the earthquake and how much occurred before it and shortly after it is not known. Davison infers from the great changes in level observed by eyewitnesses that most of it probably occurred near the time of the shock.

The most startling changes in topography occurred on the bottom of Sagami Bay. In 1923, shortly after the earthquake, sounding of the bay indicated amazing changes from the results of earlier soundings taken some ten years before. There were large areas of subsidence and also of elevation. Imamura estimates that an area of 700 square kilometers was depressed, amounting to an increase of 50 cubic kilometers in the volume of the bay, while an area of 240 square kilometers was elevated, amounting to a decrease of 20 cubic kilometers. The maximum depression was about 690 feet, while the maximum elevation was about 820 feet. In general the uplift was toward the head of the bay and the depression toward its mouth. There has been some discussion as to how much of this was due to shifting of loose material and how much to tectonic deformation.

The area of the zone of great intensity was about 11,000 square miles. The total land area over which the shock was felt was almost 105,000 square miles. The earthquake was felt at least as far as 400 miles from its center, so the total disturbed area may be said to be at least 500,000 square miles.

A tunami was set up by this shock, but it was great enough to do damage only in a few places at the ends of narrowing inlets. At Ito on Sagami Bay a 29-foot wave destroyed 300 houses. At Atami the height was estimated at 36 feet; 155 houses were destroyed and 60 people killed.

As usual in large shocks there were extensive landslides and disturbance of alluvium. Great landslides occurred on the slopes of Mount Tanzawa and Mount Hakone. The customary fissures appeared in river bottom land. Near Manazuru Point the soft ground was so shaken that potatoes were thrown out of the ground. Large trees sank into the soft soil until only their tops were visible. Near Chigasaki some old bridge piers buried and long forgotten emerged from the ground and now project 2 to 4 feet above the surface.

Although no foreshocks of this earthquake are on record, there was a great abundance of aftershocks. One thousand two hundred and fifty-six aftershocks were recorded on Tokyo seismographs in September. Twenty-four hours after the main shock there occurred another very strong earthquake[1].

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Part Two
SEISMOGRAPHY

CHAPTER VIII

The Seismograph

Operational formulas

The operational method of solution of differential equations is convenient in dealing with the equations of the seismograph. Certain rules of the operational calculus will be useful to us. First we define Heaviside's function, $H(t)$. This function has the value 0 for $t < 0$ and the value 1 for $t > 0$. Exactly at $t = 0$ it has the value 0. We shall write for the operator d/dt, the symbol p. We note that

$$pe^{\alpha t} - \alpha e^{\alpha t} = (p - \alpha)e^{\alpha t} = 0 = pH(t)$$

in the range from $t = \eta$ (η is infinitesimal) to t, the range in which we are interested.

We may then write

$$e^{\alpha t} = \frac{p}{p - \alpha} H(t), \tag{I}$$

understanding by it that if we have the equation

$$p\psi - \alpha\psi = pH(t),$$

then $\psi = e^{\alpha t}$ is a solution in the range $t = \eta$ to t. $H(t)$ is frequently not written after the operator but is to be understood.

$$\frac{\alpha}{p - \alpha} = \frac{p}{p - \alpha} - 1 = e^{\alpha t} - 1. \tag{II}$$

By a rule of algebra

$$\frac{f(t)}{F(t)} = \frac{f(a)}{F'(a)} \frac{1}{t - a} + \frac{f(b)}{F'(b)} \frac{1}{t - b} + \cdots,$$

where $f(t)$ is of the same degree as or lower degree than $F(t)$; $a, b \ldots$ are zeros of $F(t)$, are distinct and non-zero, and

$$F'(t) = \frac{d}{dt}F(t).$$

We may have in our solutions

$$\frac{f(p)}{F(p)}$$

which we may then write as

$$\frac{f(p)}{F(p)} = \sum_{\alpha} \frac{f(\alpha)}{F'(\alpha)} \frac{1}{p - \alpha},$$

where the α's are zeros of $F(p)$. From II,

$$\frac{f(p)}{F(p)} = \sum_{\alpha} \frac{f(\alpha)}{\alpha F'(\alpha)} (e^{\alpha t} - 1)$$

$$= -\sum_{\alpha} \frac{f(\alpha)}{\alpha F'(\alpha)} + \sum_{\alpha} \frac{f(\alpha)}{\alpha F'(\alpha)} e^{\alpha t}.$$

But

$$-\sum_{\alpha} \frac{f(\alpha)}{\alpha F'(\alpha)} = \sum_{\alpha} \frac{f(\alpha)}{F'(\alpha)} \frac{1}{0 - \alpha} = \frac{f(0)}{F(0)}.$$

Hence

$$\frac{f(p)}{F(p)} = \frac{f(0)}{F(0)} + \sum_{\alpha} \frac{f(\alpha)}{\alpha F'(\alpha)} e^{\alpha t}. \qquad \text{(III)}$$

Equation III is the partial fraction rule. $f(p)$ should be of the same degree as or lower degree than $F(p)$.

Consider the equation

$$\frac{d\psi}{dt} - \alpha\psi = W, \qquad \text{(IV)}$$

where W is the function of t. We write it in operational form as

$$(p - \alpha)\psi = W,$$

and obtain the formal solution

$$\psi = \frac{W}{p - \alpha}.$$

Now a solution of IV is

$$\psi = e^{\alpha t} \int_0^t W e^{-\alpha t}\, dt.$$

Therefore we may interpret

$$\frac{1}{p - \alpha} W = e^{\alpha t} \int_0^t W e^{-\alpha t}\, dt. \tag{V}$$

General theory of the seismograph

Almost all seismometers are pendulums of some kind. Pendulums are usually observed in motion, with the earth stationary. But for recording earthquakes the conditions are reversed. When the earth moves quickly, the pendulum bob tends to remain stationary while the earth moves. If the earth moves very quickly, the bob will remain almost still. If the earth moves very slowly, the bob will follow it. For intermediate earth motions the pendulum lags behind or sometimes leads the earth motion. As long as the earth motion is different from the pendulum motion, a record of this difference may be written on a rotating drum. This record is a *seismogram.*

When the earth is quiet, the pendulum is in equilibrium in the gravitational field of the earth. If the earth is displaced, two forces act on the pendulum. One is a restoring force, which tends to move the pendulum bob into a new position in gravitational equilibrium with the new position of its support, which is anchored to the moving earth; for small displacements this force is proportional to the displacement of the bob relative to the new position of rest. The second force, which opposes the first, is the inertia force of the bob; this is proportional to the acceleration of the pendulum. Let ξ denote the displacement of the bob

relative to its support (see Figure 27) and y the displacement of the earth; we have

$$A\xi + B\frac{d^2(y+\xi)}{dt^2} = 0,$$

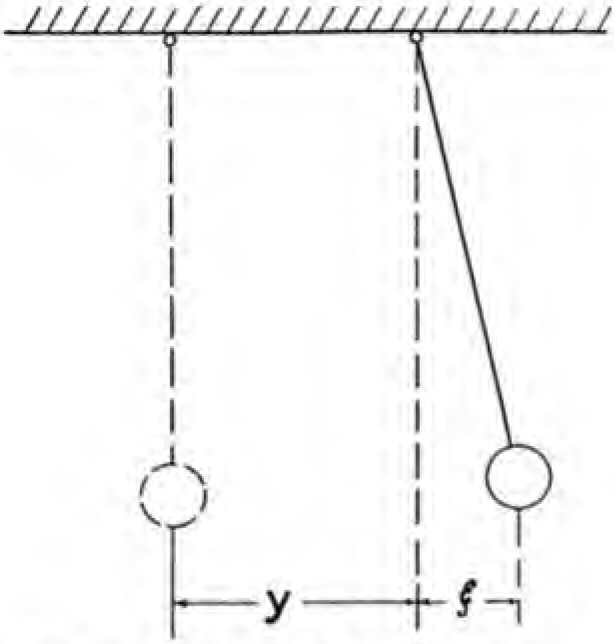

Fig. 27. Pendulum on Moving Support.

where A and B are constants. This equation expresses the fact that the summation of all forces acting on the pendulum is zero.

Whence

$$\frac{d^2\xi}{dt^2} + n^2\xi = -\frac{d^2y}{dt^2} \text{ where } n^2 \text{ is written for } \frac{A}{B}. \qquad (1)$$

Equation 1 states that the acceleration of the pendulum bob relative to its support is (a) proportional to its displacement relative to its support and oppositely directed and (b) proportional to the acceleration of the earth and oppositely directed. The sign of the right-hand member is to a certain extent arbitrary, and many writers use a plus sign. The minus holds to the convention that if a displacement of the earth in a given direction is positive, then

a displacement of the pendulum, relative to its support, in that direction is also positive. This relation may be seen from an examination of the case where $n^2 = 0$ (no restoring force, a pendulum of infinite period). Then the mass will not move, and if the earth moves north, the apparent movement of the mass relative to its support is south. Equation 1 becomes

$$\frac{d^2\xi}{dt^2} = -\frac{d^2y}{dt^2};$$

that is, we measure y as positive and ξ as negative in this case.

In order to avoid resonance (unduly large amplitudes of motion when the earth moves with a period near that of the free period of the pendulum), it is customary to introduce damping. For mathematical simplicity it is well to attempt a damping proportional to the speed of the pendulum with respect to its support. To provide for damping we may note that the addition of a damping mechanism provides an additional acceleration which is proportional to $d\xi/dt$ and which is oppositely directed to, that is, opposes, $d^2\xi/dt^2$. Then 1 becomes

$$\frac{d^2\xi}{dt^2} + 2\kappa\frac{d\xi}{dt} + n^2\xi = -\frac{d^2y}{dt^2},$$

where we have taken 2κ as the proportionality factor in the damping term.

If the pendulum is equipped for recording, the displacement, x, on the record will be larger than the displacement of the mass, ξ, by a magnification factor V. Putting $x = V\xi$ and setting the earth acceleration $d^2y/dt^2 = \Phi$ for brevity, we have

$$\frac{d^2x}{dt^2} + 2\kappa\frac{dx}{dt} + n^2x = -V\Phi. \tag{2}$$

Putting this equation into operational form and taking care of initial conditions, we get

$$p[p(x - x_0) - (px)_0] + 2\kappa p(x - x_0) + n^2 x = -V\Phi,$$

where the zero subscripts indicate values at $t = 0$. Putting $(px)_0 = v_0$, the initial speed of the recording element (pen point or light spot) with respect to the recording drum,

$$p^2 x + 2\kappa p x + n^2 x = (p^2 + 2\kappa p)x_0 + p v_0 - V\Phi.$$

$$x = \frac{(p^2 + 2\kappa p)x_0 + p v_0 - V\Phi}{p^2 + 2\kappa p + n^2}. \tag{3}$$

Write

$$p^2 + 2\kappa p + n^2 = (p - \alpha)(p - \beta).$$

Then

$$\left.\begin{aligned} \alpha &= -\kappa \mp \sqrt{\kappa^2 - n^2} \\ \beta &= -\kappa \pm \sqrt{\kappa^2 - n^2}. \end{aligned}\right\} \tag{4}$$

We must evaluate

$$x = \frac{(p^2 + 2\kappa p)x_0 + p v_0}{(p - \alpha)(p - \beta)} - \frac{V\Phi}{(p - \alpha)(p - \beta)}. \tag{5}$$

Call the first term x_a,

$$x_a = p\left\{\frac{(p - \alpha - \beta)x_0 + v_0}{p^2 - (\alpha + \beta)p + \alpha\beta}\right\}.$$

Apply the partial fraction rule to the term in brackets (formula III), and then operate with $p = d/dt$.

$$x_a = \frac{(v_0 - \beta x_0)e^{\alpha t} - (v_0 - \alpha x_0)e^{\beta t}}{\alpha - \beta} \tag{6}$$

To evaluate the second term of the right-hand member of 5 (call it x_b),

$$x_b = \frac{-V\Phi}{(p - \alpha)(p - \beta)} = \frac{-V}{\alpha - \beta}\left\{\frac{\Phi}{p - \alpha} - \frac{\Phi}{p - \beta}\right\}. \tag{7}$$

From formula V

$$x_b = \frac{-V}{\alpha - \beta}\left\{e^{\alpha t}\int_0^t \Phi e^{-\alpha t}\,dt - e^{\beta t}\int_0^t \Phi e^{-\beta t}\,dt\right\}.$$

Now

$$x = x_a + x_b.$$

Therefore

$$x = \frac{1}{\alpha - \beta}\Big\{(v_0 - \beta x_0)e^{\alpha t} - (v_0 - \alpha x_0)e^{\beta t} - Ve^{\alpha t}\int_0^t \Phi e^{-\alpha t}\,dt$$
$$+ Ve^{\beta t}\int_0^t \Phi e^{-\beta t}\,dt\Big\},$$

where α and β are defined by equations 4. This is a general equation of the response of a seismograph to an earth acceleration of Φ. It must be examined later for particular types of Φ.

Earth quiet. The behavior of the pendulum may be investigated by giving it an impulse and allowing it to swing while the earth is quiet. This impulse is equivalent to giving the earth a speed $CH(t)$, where $H(t)$ is Heaviside's function. The application to the earth at time $t = 0$ of a constant speed C which is not altered thereafter amounts to the same thing as giving the pendulum a very large instantaneous acceleration at $t = 0$.

We then set

$$\Phi = CpH(t).$$

Now in this case the velocity of the earth vanishes at $t = 0$, and therefore $v_0 = 0$ and also $x_0 = 0$.

Then from 7,

$$x = -\frac{VC}{\alpha - \beta}\left(\frac{p}{p - \alpha}H(t) - \frac{p}{p - \beta}H(t)\right),$$

and from formula I

$$x = \frac{VC}{\alpha - \beta}(e^{\beta t} - e^{\alpha t}).$$

From 4

$$x = \frac{-VC}{2\sqrt{\kappa^2 - n^2}}e^{-\kappa t}(e^{\sqrt{\kappa^2 - n^2}t} - e^{-\sqrt{\kappa^2 - n^2}t})$$
$$= \frac{-VC}{\sqrt{\kappa^2 - n^2}}e^{-\kappa t}\sinh\sqrt{\kappa^2 - n^2}\,t.$$
$$x = \frac{-VC}{\sqrt{n^2 - \kappa^2}}e^{-\kappa t}\sin\sqrt{n^2 - \kappa^2}\,t. \tag{8}$$

If $\kappa = 0$, that is, the pendulum is undamped,

$$x = -\frac{CV}{n} \sin nt;$$

that is, the free period of the pendulum is

$$T_0 = \frac{2\pi}{n}.$$

The damped period is

$$T_0' = \frac{2\pi}{\sqrt{n^2 - \kappa^2}}.$$

$$T_0 = \frac{T_0'}{\sqrt{1 + \left(\frac{\kappa T_0'}{2\pi}\right)^2}}. \qquad (9)$$

If $\kappa = n$, we have critical damping or aperiodicity. This is the minimum value of κ for which the pendulum does not cross its line of rest on a free swing. T_0' becomes infinite. For $\kappa > n$ the pendulum is overdamped, and it writes a hyperbolic sine curve.

If the damped pendulum is allowed to swing free (damping less than critical) and two successive crests are measured on the record and the ratio of their amplitudes without regard to sign is called ϵ, then

$$\epsilon = \frac{x_1}{-x_2} = \frac{e^{-\kappa t_1}}{e^{-\kappa t_2}} = e^{\kappa(t_2 - t_1)} = e^{\kappa \frac{T_0'}{2}}.$$

$$\log_e \epsilon = \frac{\kappa T_0'}{2}.$$

Combining with 9,

$$\left(\frac{\kappa T_0}{2\pi}\right)^2 = \frac{\frac{\log_e^2 \epsilon}{\pi^2}}{1 + \frac{\log_e^2 \epsilon}{\pi^2}} = \frac{\log_{10}^2 \epsilon}{1.862 + \log_{10}^2 \epsilon}. \qquad (10)$$

ϵ is the damping ratio as usually supplied in seismographic bulletins. Equation 10 will be found useful later.

The simple pendulum. A simple pendulum is an ideal, one whose mass is entirely concentrated at a point. The restoring force is $Mg \sin \theta$ (see figure 28), where M is the mass, g the acceleration of gravity, and θ the angular displacement. If the angle is small, the restoring force is very closely $Mg \frac{\xi}{l}$ where ξ is the linear displacement of the mass and l the length of the pendulum. The inertia force is $M \frac{d^2\xi}{dt^2}$.

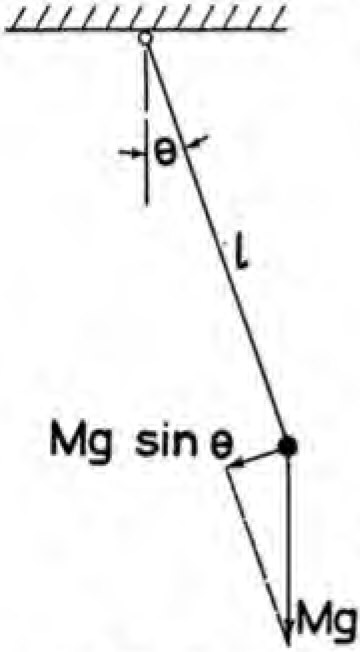

Fig. 28. Simple Pendulum.

Equation 1 then becomes

$$M \frac{d^2\xi}{dt^2} + Mg \frac{\xi}{l} = -M \frac{d^2y}{dt^2}$$

$$\frac{d^2\xi}{dt^2} + \frac{g}{l} \xi = -\frac{d^2y}{dt^2}$$

$$n^2 = \frac{g}{l}.$$

$$T_0 = 2\pi \sqrt{\frac{l}{g}}. \qquad (11)$$

Equation 11 is the well known equation for the free period of a simple pendulum.

The physical pendulum. An actual pendulum does not have its mass concentrated at a point. For it we must write the equation of force moments about its center of suspension. Let h be the distance from the center of mass C to the center of suspension S. The restoring force $Mg \sin \theta$ acts at the center of mass, and this force is again approximately $Mg\theta$ for small displacements. The restoring force moment is $Mgh\theta$. The inertial force moment (earth quiet) is $K(d^2\theta/dt^2)$, where K is the moment of inertia about S. Therefore

$$K \frac{d^2\theta}{dt^2} + Mgh\theta = 0.$$

$$\frac{d^2\theta}{dt^2} + \frac{Mgh}{K} \theta = 0.$$

Hence the free period is

$$T_0 = 2\pi\sqrt{\frac{K}{Mgh}}.$$

We may write

$$K = M(k^2 + h^2),$$

where k is the radius of gyration about the center of mass.

$$T_0 = \sqrt{\frac{k^2 + h^2}{gh}}. \tag{12}$$

By comparing 11 and 12 we see that a simple pendulum of the same free period as the compound pendulum would have

$$l = \frac{k^2 + h^2}{h}$$

$$h(l - h) = k^2 \tag{13}$$

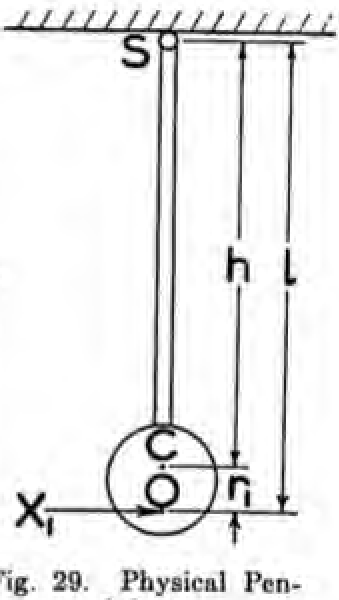

Fig. 29. Physical Pendulum.

So l is greater than h and is at some point as O in Figure 29. O is called the *center of oscillation* of the pendulum. It is the point at which the mass of the physical pendulum could be concentrated and leave the position of rest and the free period unaltered. Suppose the pendulum is suspended at O. In 13, k^2 remains unchanged as it is a physical constant of the pendulum. $l - h$ becomes the new h, and therefore the old h becomes the new $l - h$. So l is unchanged, and the old center of suspension becomes the new center of oscillation. The center of suspension and the center of oscillation are interchangeable.

Again, referring to Figure 29, if the pendulum and support are rigid up to the center of suspension S, we know that

$$\Sigma X dt = M d\bar{u} = M h d\omega$$

approximately, if displacements are quite small. Here the Xdt's denote impulses in the x direction acting on the

pendulum, $\bar{u}$ the velocity of the center of mass C in the x direction, and ω the angular velocity about S. Furthermore

$$\Sigma X r dt = K d\omega = M k^2 d\omega,$$

where r is the normal distance from the center of mass to the line of application of the corresponding X. Now if X is applied at *the center of percussion*, there will be no impulse in the x direction at S. Apply only one force, X_1, at a point on axis SC at a distance, r_1, below C. Then if this point of application is at the center of percussion, X_1 is the only force acting on the system in the x direction and

$$X_1 dt = M h d\omega$$

and

$$X_1 r_1 dt = M k^2 d\omega.$$

Therefore

$$h r_1 = k^2.$$

Whence from equation 13

$$r_1 = l - h,$$

and the center of percussion is at the center of oscillation. Therefore the center of percussion and the center of suspension are interchangeable. A blow struck at the center of suspension by a sudden movement of the earth in the x direction causes no force in that direction at the center of percussion. This point then does not move until the displacement of suspension brings into play gravitational forces of restoration. Thus fast earthquake motion (motion of a period much shorter than the free period of the pendulum) effectively interchanges the center of oscillation and center of suspension. The center of suspension moves rapidly, carried with the earth. It is the center of oscillation then which remains stationary. The "steady point" of our pendulum is its center of oscillation. In the theory above we spoke of ξ as the displacement of the mass. Actually it is the displacement of the center of oscillation, for all parts of the mass are not displaced by the same amount.

Horizontal pendulums. It is clear from formula 12 that to get a long free period it is necessary to make h either very large or very small. A very long pendulum hanging vertically would be cumbersome. Schlüter built a seismograph of very small h, but such instruments are not used today. Long-period seismographs are usually built in the

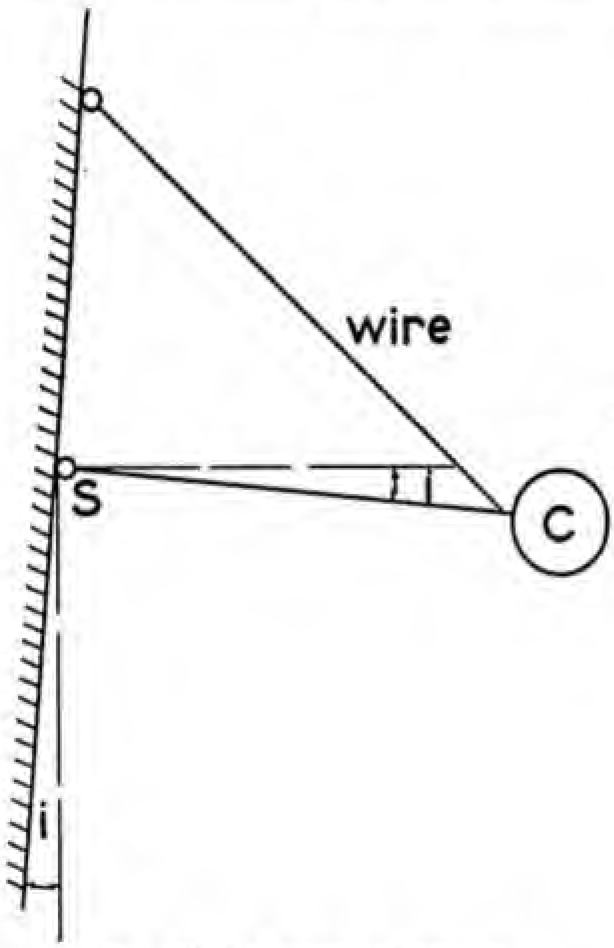

Fig. 30. Horizontal Pendulum.

form of horizontal pendulums. The boom connecting the center of suspension S (see Figure 30) and the center of mass C is held almost horizontal. The effective component of gravity becomes $Mg \sin i$, where i is the angle the axis of rotation makes with the vertical. The restoring force moment becomes $Mhg\theta \sin i$, and the free period

$$T_0 = \sqrt{\frac{K}{Mhg \sin i}},$$

and for i very small

$$T_0 = \sqrt{\frac{K}{Mhgi}}.$$

A simple pendulum so tilted would have

$$T_0 = \sqrt{\frac{l}{gi}}.$$

So the distance of the hinge to the center of oscillation of a horizontal pendulum is still

$$l = \frac{K}{Mh}.$$

Vertical motion pendulums. Vertical motion pendulums are of two types. In the first the mass is suspended by a spring so that the center of mass is in the same vertical line as the center of suspension. Here the restoring force is $P\xi$ where P is the spring constant in terms of force per unit stretch. (The gravitational force is balanced by the initial tension in the spring.) The inertia force is, as usual, $M\frac{d^2(\xi + y)}{dt^2}$; so

$$\frac{d^2\xi}{dt^2} + \frac{P}{M}\xi = -\frac{d^2y}{dt^2}$$

$$T_0 = 2\pi\sqrt{\frac{M}{P}}.$$

To get a longer period instrument the vertical is usually built as in Figure 31 with a hinge at S and center of mass C (mass at rest) in the same horizontal plane as S. This position is necessary to eliminate recording of horizontal vibrations. The spring is generally attached below line SC and at a point between S and C.

In developing the theory we follow Schnirman[1]. Take the origin of coordinates at S (see Figure 31). Let the x axis lie along the line joining S and the point of attachment of the lower end of the spring, pendulum at rest. The y axis is at right angles to the x axis. x_1 y_1 are the variable coordinates of the lower end of the spring; x_2 y_2 are the

fixed coordinates of the upper end of the spring. At rest $x_1 = r$, $y_1 = 0$. Let A equal initial length of the spring, that is, its length without the mass of the pendulum

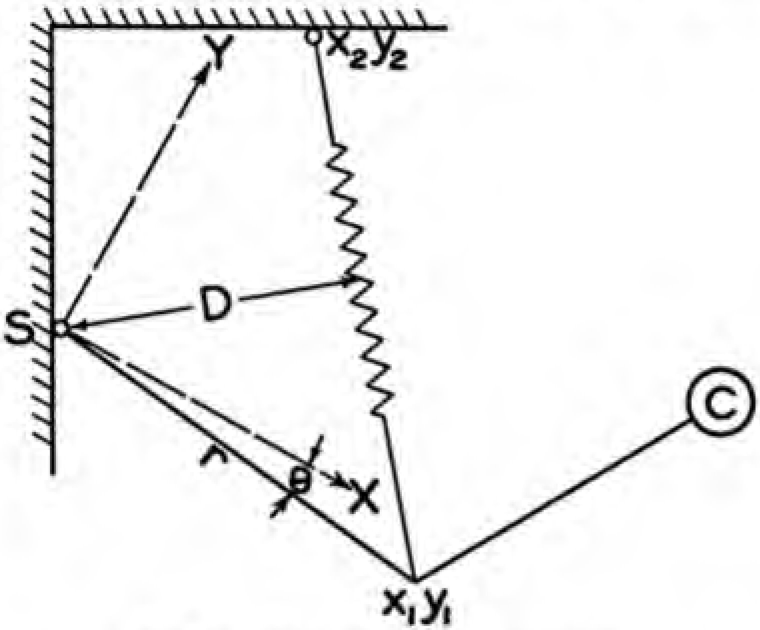

Fig. 31. Vertical Motion Pendulum.

attached. Then $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} - A$ is the stretch of the spring, and the force of the spring is

$$f = P(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} - A).$$

Now the lever arm of the spring about S is

$$D = \frac{x_1y_2 - y_1x_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}.$$

The force moment of the spring is

$$fD = Pr(y_2 \cos \theta - x_2 \sin \theta) \left(1 - \frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2r(x_2 \cos \theta + y_2 \sin \theta)}}\right), \quad (14)$$

where we have set

$$x_1 = r \cos \theta$$
$$y_1 = r \sin \theta,$$

θ being the angular displacement of the pendulum. Now the gravitational force moment tending to displace the

pendulum down is $-Mhg \cos \theta$, where h is the distance from S to C, the center of mass.

When the pendulum is at rest at $\theta = 0$, the gravitational force moment is equal to the spring force moment and is of opposite sign, so

$$-Mhg = -Pry_2\left[1 - \frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2rx_2}}\right].$$

$$-Mhg \cos \theta = -Pry_2 \cos \theta \left[1 - \frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2rx_2}}\right], \quad (15)$$

and the total force moment on the pendulum is from 14 and 15.

$$\begin{aligned} F &= fD - Mhg \cos \theta \qquad (16) \\ &= +Pry_2 \cos \theta \left(\frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2rx_2}} - \frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2r(x_2 \cos \theta + y_2 \sin \theta)}}\right) \\ &\quad -Prx_2 \sin \theta \left(1 - \frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2r(x_2 \cos \theta + y_2 \sin \theta)}}\right). \end{aligned}$$

We see in equation 16 a difficulty in the vertical component seismograph. The terms in $\cos \theta$ and $\sin \theta$ may be expanded for small values of θ. Now the $\sin \theta$ expansion terms have only odd powers of θ, and each of these changes sign with θ, giving equal increments to the restoring force whether θ is positive or negative. But the $\cos \theta$ expansion has even powers of θ, and their contribution to the restoring force is in the same direction on either side of rest and therefore makes for lack of symmetry and unbalance. The equations for the horizontal component pendulums had no $\cos \theta$ terms.

From 16 we see that if $A = 0$ we get a force moment of only

$$-Prx_2 \sin \theta.$$

The equation of such a pendulum for earth quiet becomes

$$K \frac{d^2\theta}{dt^2} + Prx_2 \sin \theta = 0,$$

where K is the moment of inertia about the hinge. For small values of θ

$$T_0 = 2\pi \sqrt{\frac{K}{Prx_2}}.$$

If x_2 is made 0, $T_0 = \infty$. This means attaching the upper end of the spring to a point on the y axis, that is, on a line perpendicular to arm r at S (pendulum at rest). In order to maintain SC horizontal in the rest position it is further necessary from 15 that

$$y_2 = \frac{Mgh}{Pr},$$

that is, the upper spring connection be fastened on the y axis at a distance $\dfrac{Mgh}{Pr}$ from S.

It is possible to wind a spring of initial zero length. A great advantage of such a spring is the elimination of the term in $\cos\theta$ from the restoring force and therefore a symmetrical response.

A very long period pendulum is one of very small restoring force and one therefore very sensitive to many extraneous influences (temperature in particular) which we do not wish to record.

Returning to equation 16, if θ is very small, the restoring force moment becomes to first orders in θ

$$F = Pry_2 \frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2rx_2}} \left(1 - \frac{1}{\sqrt{1 - \dfrac{2ry_2\theta}{r^2 + x_2^2 + y_2^2 - 2rx_2}}}\right)$$

$$-Prx_2\theta \left(1 - \frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2rx_2}} \frac{1}{\sqrt{1 - \dfrac{2ry_2\theta}{r^2 + x_2^2 + y_2^2 - 2rx_2}}}\right)$$

$$F = -Prx_2\theta \left(1 - \frac{A}{\sqrt{r^2 + x_2^2 + y_2^2 - 2rx_2}} + \frac{Ary_2^2}{x_2(r^2 + x_2^2 + y_2^2 - 2rx_2)^{\frac{3}{2}}}\right). \quad (16a)$$

The differential equation of the pendulum is then, remembering equation 15, ground at rest,

$$K\frac{d^2\theta}{dt^2} + Prx_2\theta\left[\frac{Mhg}{Pry_2} + \frac{ry_2^2}{A^2x_2}\left(1 - \frac{Mhg}{Pry_2}\right)^3\right] = 0.$$

$$T_0 = 2\pi\sqrt{\frac{K}{Pr\left[\frac{Mhgx_2}{Pry_2} + \frac{r}{A^2y_2}\left(y_2 - \frac{Mhg}{Pr}\right)^3\right]}}.$$

$$T_0 = 2\pi\sqrt{\frac{K}{Mgh\left(\tan\alpha + \frac{MghA}{LP(L-A)^2}\right)}}, \tag{17}$$

where L is the length of the spring when the pendulum is at rest, and $\alpha = \tan^{-1}(x_2/y_2)$.

The force moment acting on the pendulum to second orders in θ may be obtained from equation 16 by expansion. The contribution of θ^2 is

$$+\frac{3}{2}Pr^2y_2A(x_2 - r)\frac{x_2(x_2 - r) + y_2^2}{(r^2 + x_2^2 + y_2^2 - 2rx_2)^{\frac{3}{2}}}\theta^2.$$

This undesirable second-order term may be made to vanish by setting $x_2 = r$, that is, by having the spring perpendicular to the boom r at rest. Early vertical seismometers were so designed, the spring hanging vertically and the boom being horizontal. But when $x_2 = r$, the multiplier of the term in θ (see equation 16a) reduces to $-Pr^2$, which means that a shorter period is the result of this effort to eliminate the term in θ^2; so later seismometers sacrificed symmetry and allowed the distortion of the θ^2 term in order to obtain a longer free period. Auxiliary springs are used in some vertical instruments (Wilip-Galitzin) to offset the unsymmetry.

Determination of constants. The constants in equation 2, the general equation of the seismograph, are three: κ, n^2, V. Now

$$n^2 = \left(\frac{2\pi}{T_0}\right)^2,$$

where T_0 is the free period of the pendulum. To determine T_0 remove the damping, allow the instrument to swing free, and determine the time of a complete swing. This is T_0. Restore the damping and allow the pendulum to execute damped vibration. The ratio of two successive amplitudes (one up, one down) is the damping ratio ϵ. For a well damped instrument there will be only two crests. The damping ratio ϵ is connected with κ by equation 10. The static magnification V is the ratio of the displacement of the writing indicator to the displacement of the center of oscillation, that is,

$$V = \frac{x}{l\theta}.$$

It is simple to measure x, the amplitude on the record corresponding to any angular deflection θ of the mass. θ is measured by a mirror and scale, the mirror frequently being that on the mass used for recording and the scale frequently being the record itself. l, the distance from the hinge to the center of oscillation, is a constant whose value is frequently supplied by the instrument builders. For instruments of simple, symmetrical design as the Wood-Anderson it may be computed. If it is not readily available, it may be obtained by one of two methods:

I. Upset method. The pendulum may be "upset" to make it hang vertically. This may necessitate some ingenuity, since in the horizontal pendulum the hinge is a compression joint, whereas in the vertical position it is a tension joint. Cords may be used to hold it in place. In the new vertical position the new free period may be measured:

$$T_0 = 2\pi\sqrt{\frac{l}{g}},$$

and l computed. The method may be used on the vertical seismograph also.

II. Tilt method. If the instrument is a horizontal pendulum, it may be tilted through a small angle ψ about

a horizontal axis which lies in the vertical plane containing the hinge and the center of mass; the accompanying angular deflection θ of the mass will equal approximately $\frac{\psi}{i}$, where i is the angle the axis of rotation of the pendulum makes with the vertical. Now

$$T_0 = 2\pi\sqrt{\frac{l}{ig}},$$

and with i known from the tilt test and T_0 from direct observation, l may be computed.

In some seismometers with electromagnetic damping the magnetic field introduces a restoring force as well as a damping force. This force is due to magnetic impurities in the damping vanes, and to tilting of the base by the weight of the magnets altering the effective component of gravity. In such cases the free period measured with the magnets removed is not the effective free period when the magnets are restored. If the value of l is known otherwise, then the value of i determined in the tilting test may be used to determine the effective free period from

$$T_0 = 2\pi\sqrt{\frac{l}{gi}}.$$

If the position of the center of mass of the pendulum system is known (if h is known), a given force moment Z may be applied to the pendulum boom to produce a displacement x of the pen. If the instrument is a horizontal pendulum,

$$Z = Mg\theta h \sin i,$$

$$\theta = \frac{Z}{Mgh \sin i},$$

$$l\theta = \frac{Z}{Mgh \sin i}\,\frac{gT_0^2 \sin i}{4\pi^2},$$

and

$$V = \frac{x}{l\theta} = \frac{4\pi^2 Mh}{ZT_0^2}\,x.$$

Dynamic magnification. In equation 5 we have the solution of the indicator displacement x in terms of the earth acceleration Φ. Suppose

$$\Phi = \frac{d^2y}{dt^2} = A \sin \omega t.$$

Then the earth velocity is

$$\frac{dy}{dt} = -\frac{A}{\omega} \cos \omega t[+C_1 = 0].$$

C_1 must be zero, since we want the earth displacement to be simple harmonic. The earth displacement is then

$$y = -\frac{A}{\omega^2} \sin \omega t[+C_2 = 0].$$

We let $C_2 = 0$, since we wish earth displacement to be zero, as well as earth acceleration, when $t = 0$. Since sine and cosine functions do not begin or end, that is, values exist for them for all values of t, an assumption of these initial conditions amounts to putting the seismograph onto a pier executing simple harmonic motion at a moment when displacement and acceleration are zero and velocity is a maximum, $-\dfrac{A}{\omega}$.

Hence our initial conditions for substitution in equation 4 are

$$\frac{x_0}{V} = -y_0 = 0,$$

$$\frac{v_0}{V} = -\left(\frac{dy}{dt}\right)_0 = \frac{A}{\omega}.$$

For at the moment at which the pier motion begins, the center of oscillation of the pendulum is stationary with respect to the earth as a whole.

We must now evaluate the second part of equation 5:

$$x_b = -\frac{VA \sin \omega t}{(p-\alpha)(p-\beta)}$$
$$= \frac{VA}{\alpha - \beta}\left\{\frac{\sin \omega t}{p-\beta} - \frac{\sin \omega t}{p-\alpha}\right\},$$

which, from formula V, becomes

$$x_b = \frac{VA}{\alpha - \beta}\left\{e^{\beta t}\int_0^t e^{-\beta t}\sin\omega t\,dt - e^{\alpha t}\int_0^t e^{-\alpha t}\sin\omega t\,dt\right\}$$
$$= \frac{VA}{\alpha - \beta}\left\{\frac{-\beta\sin\omega t - \omega\cos\omega t}{\beta^2 + \omega^2} + e^{\beta t}\frac{\omega}{\beta^2 + \omega^2}\right.$$
$$\left. + \frac{\alpha\sin\omega t + \omega\cos\omega t}{\alpha^2 + \omega^2} - e^{\alpha t}\frac{\omega}{\alpha^2 + \omega^2}\right\}.$$

The first part of solution 5 is

$$x_a = \frac{VA}{(\alpha - \beta)\omega}(e^{\alpha t} - e^{\beta t});$$

so the solution is

$$x = x_a + x_b.$$

$$x = \frac{VA}{\omega(\alpha - \beta)}\left[e^{\alpha t}\frac{\alpha^2}{\alpha^2 + \omega^2} - e^{\beta t}\frac{\beta^2}{\beta^2 + \omega^2}\right]$$
$$+ \frac{VA}{\alpha - \beta}\left[\left(\frac{\alpha}{\alpha^2 + \omega^2} - \frac{\beta}{\beta^2 + \omega^2}\right)\sin\omega t\right.$$
$$\left. + \left(\frac{\omega}{\alpha^2 + \omega^2} - \frac{\omega}{\beta^2 + \omega^2}\right)\cos\omega t\right]. \quad (18)$$

Now

$$\alpha = -\kappa - \sqrt{\kappa^2 - n^2};$$
$$\beta = -\kappa + \sqrt{\kappa^2 - n^2}.$$

Hence the terms in $e^{\alpha t}$ and $e^{\beta t}$ die off with time if there is damping, that is, $\kappa \neq 0$. Considering the transient portion of the response if $\kappa = n = -\alpha = -\beta$ (critical damping), the transient becomes

$$x_t = \frac{-2\omega\kappa VA}{(\kappa^2 + \omega^2)^2}e^{-\kappa t},$$

and remembering that

$$\omega = \frac{2\pi}{T},$$
$$n = \frac{2\pi}{T_0},$$

the transient is

$$x_t = \frac{-VAT_0^3T^3}{2\pi^2[T^2 + T_0^2]^2}e^{-\frac{2\pi}{T_0}t}. \quad (18a)$$

After the first transient terms have died out,

$$x = \frac{VA}{(\alpha^2 + \omega^2)(\beta^2 + \omega^2)} \{(\omega^2 - \alpha\beta) \sin \omega t - \omega(\alpha + \beta) \cos \omega t\}. \tag{18b}$$

Put

$$\omega^2 - \alpha\beta = J \cos \gamma,$$

and

$$\omega(\alpha + \beta) = J \sin \gamma.$$

Then

$$\tan \gamma = \frac{\omega(\alpha + \beta)}{\omega^2 - \alpha\beta} = \frac{1}{\pi} \frac{KT_0^2 T}{T^2 - T_0^2},$$

T and T_0 being respectively the period of earth motion and free period of seismograph.

Now

$$J^2 = (\omega^2 - \alpha\beta)^2 + \omega^2(\alpha + \beta)^2.$$

Substituting in 18b,

$$x = \frac{V\omega^2 \sqrt{(\omega^2 - \alpha\beta)^2 + \omega^2(\alpha + \beta)^2}}{(\alpha^2 + \omega^2)(\beta^2 + \omega^2)} \left\{\frac{A}{\omega^2} \sin (\omega t - \gamma)\right\}.$$

So *after the transient has died out*, x, the indicator response, varies with the earth displacement, is of opposite sign to it, and lags by the angle γ, by time γ/ω.

The constant by which the amplitude of ground motion is multiplied is called the *dynamic magnification* and is

$$\vartheta = \frac{V\omega^2 \sqrt{(\omega^2 - \alpha\beta)^2 + \omega^2(\alpha + \beta)^2}}{(\alpha^2 + \omega^2)(\beta^2 + \omega^2)}$$

$$= \frac{V}{\sqrt{\left[1 - \left(\frac{T}{T_0}\right)^2\right]^2 + \frac{\kappa^2 T^2}{\pi^2}}},$$

which from 10 becomes

$$\vartheta = \frac{V}{\sqrt{\left[1 - \left(\frac{T}{T_0}\right)^2\right]^2 + 4 \frac{\log_{10}^2 \epsilon}{1.862 + \log_{10}^2 \epsilon} \left(\frac{T}{T_0}\right)^2}}. \tag{19}$$

Let us investigate equation 19:

A. The case in which $T_0 \ll T$, that is, the free period of the pendulum is much less than that of the earth motion:

$$\begin{aligned}
\vartheta &\doteq \left(\frac{T_0}{T}\right)^2 V. \\
x &= \vartheta \frac{A}{\omega^2} \sin(\omega t - \gamma), \\
&= \frac{T_0^2}{T^2} V \frac{A}{\left(\frac{2\pi}{T}\right)^2} \sin(\omega t - \gamma), \\
&= \frac{T_0^2 V}{(2\pi)^2} A \sin(\omega t - \gamma).
\end{aligned}$$

Therefore, after the transient has died out, the seismogram displacement is proportional to the ground acceleration, is of same sign, and lags by an angle

$$\gamma = \tan^{-1} \frac{\kappa T_0^2}{T} \text{ (approximately, since } T \gg T_0),$$

which is a positive angle. Investigating the transient for this case and critical damping, we get for 18a:

$$x_t = -\left[\frac{2T_0}{T} e^{-\frac{2\pi}{T_0}t}\right] \frac{T_0^2 V}{(2\pi)^2} A.$$

Even when $t = 0$, the term in brackets is very small, and it multiplies a term which is equal to the amplitude of the harmonic response.

A seismometer of free period T_0 much less than the period of any wave it records may be called an *accelerometer*.

B. The case in which $T_0 \gg T$:

$$\vartheta \doteq V$$

and

$$x = V \frac{A}{\omega^2} \sin(\omega t - \gamma).$$

The harmonic response is proportional to earth displacement, is of opposite sign, and lags by an angle $\tan^{-1}\left(-\frac{\kappa T}{\pi}\right)$

approximately; that is, it leads. An instrument of free period longer than the period of any wave it records may be called a *displacement meter*. In this case the transient from 18a (critical damping) becomes

$$x_t = -\left[\frac{2T}{T_0} e^{-\frac{2\pi}{T_0}t}\right] \frac{VA}{\omega^2}.$$

Here again even at $t = 0$, the value of the bracketed term is very small, so that the greatest value of x_t is much less than the amplitude of the harmonic response.

C. The case of $T_0 \doteq T$ (not exactly equal):

$$\vartheta \doteq \frac{VT_0}{T\sqrt{4\dfrac{\log_{10}^2 \epsilon}{1.862 + \log_{10}^2 \epsilon}}},$$

and

$$x = \frac{VT_0}{2\pi\sqrt{4\dfrac{\log_{10}^2 \epsilon}{1.862 + \log_{10}^2 \epsilon}}} \frac{A}{\omega} \sin(\omega t - \gamma).$$

Here

$$\gamma = \tan^{-1} \infty = 90°.$$

So

$$x = -\frac{VT_0}{2\pi\sqrt{4\dfrac{\log_{10}^2 \epsilon}{1.862 + \log_{10}^2 \epsilon}}} \frac{A}{\omega} \cos \omega t.$$

For critical damping

$$x = -\frac{VT_0}{4\pi} \frac{A}{\omega} \cos \omega t.$$

The harmonic response is proportional to the velocity of the earth and is in phase with it. Here the transient becomes (for critical damping)

$$x_t = -e^{-\frac{2\pi}{T_0}t} \frac{VT_0}{4\pi} \frac{A}{\omega}.$$

Thus at $t = 0$ the transient is large; it has the magnitude of the harmonic amplitude. It dies off with time expo-

nentially, as does the transient in any case. When $t = 0.37T_0$, the value of x_t has decreased to one tenth the harmonic amplitude (maximum). (Remember that we are considering a fictitious shock which begins with finite velocity, that is, velocity of which is given by the cosine function.)

Actually any instrument records at various times waves of periods greater than, equal to, and less than its own free period.

It should be noted that $\tan \gamma$ has multiple roots. For simple harmonic motion there is no significance in increasing the lag by 360°, since all waves are exactly alike. Also the statement that response is opposite in sign is equivalent to increasing lag or lead by 180°. The irregularities of the seismogram indicate very frequent changes in the character of the motion. Even though some short runs may be reasonably approximated by simple harmonic motion, transient terms in the response are rarely long absent.

Electromagnetic seismographs

The great difficulty experienced in maintaining a constant zero position in a vertical seismograph of high magnification and reasonably long period (say 12 seconds) leads one to seek a method of recording in which slow drifts are neglected. Such a method, developed first by Prince Galitzin, is to record through a galvanometer. One element of an electrical transducer, in Galitzin's case the coil, is attached to the sprung mass. The other elements are fastened to the frame. Relative motion between the parts, due to an earthquake moving the frame, induces an electromotive force in the coil. This coil is connected to a galvanometer which responds to the induced electromotive force. Now the induced electromotive force is proportional to the relative velocity of mass to that of frame. Therefore slow drifts, caused by temperature changes, for instance, are not recorded. Thus the great difficulty of holding the zero constant on the seismogram is eliminated.

It is highly desirable to have the three components of a seismograph (two horizontals and a vertical) of identical constants. Then one may study the direction of motion of the earth during the passage of various types of elastic waves without the laborious task of integrating the record, or the alternative, the making of rough approximations as the basis of reduction of trace amplitude. By use of galvanometric recording it is relatively simple to give the three components similar constants. The theory of such recording is, however, not so simple as that for direct registration.

Galitzin type

We shall investigate first the seismometer in which the coil is attached to the mass[2]. Figure 32 illustrates dia-

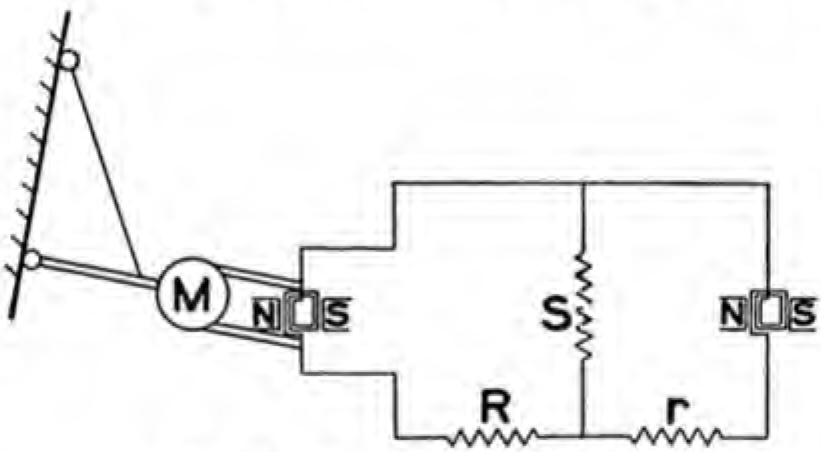

Fig. 32. Moving Coil Seismometer.

grammatically the system. The left-hand coil represents the coil of the seismometer which rotates with M in a plane at right angles to the plane of the figure. Its resistance and that of its leads is R. The right-hand coil represents the galvanometer of resistance r. This coil rotates about its own longitudinal axis. There is a shunt of resistance S.

Let φ be the angular displacement of the seismometer and θ that of the galvanometer. The electromotive force generated in a coil is proportional to the effective number of lines of force cut per unit time.

For a galvanometer consisting of a coil in a uniform parallel magnetic field the effective number of lines of force threading the coil is equal to its area times the number of turns times the field strength (which double product we shall call g) times $\sin\theta$, and the electromotive force generated is

$$e_g = -g\frac{d\theta}{dt}$$

when θ is small; g is called the electrodynamic constant of the galvanometer. The negative sign indicates that the electromotive force opposes that generated by the seismometer.

The torque acting on the galvanometer coil owing to a current, i, flowing in it is $gi \cos\theta$, and for θ small we may write

$$F_g = gi.$$

For the seismometer the electromotive force generated is (for small φ)

$$e_s = G\frac{d\varphi}{dt},$$

where G is the electrodynamic constant of the seismometer. The value of G depends on the geometry of the seismometer system. In the Wenner seismometer the magnetic field is radial to the axis of the coil and G is the product of the field strength by the number of turns by the circumference of the coil by the distance from the center of the coil to the axis of rotation. For Galitzin seismometers the coil moves between horseshoe magnets, the field being in opposite directions on the two sides of the coil which cut it. For them G is the product of twice the width of the coil by the number of turns by the field strength by the distance from the center of the coil to the axis of rotation. The torque on the seismometer due to the current I flowing in it is

$$F_s = GI.$$

If the galvanometer be clamped so that it cannot move, the current in the seismometer will be

$$(I) = \frac{G\dfrac{d\varphi}{dt}}{R + \dfrac{rS}{r+S}} = \frac{G\dfrac{d\varphi}{dt}(r+S)}{Q^2},$$

where

$$Q^2 = rR + rS + RS.$$

The current in the galvanometer is

$$(i) = \frac{GS\dfrac{d\varphi}{dt}}{Q^2}.$$

Similarly, if the seismometer is clamped and the galvanometer moves,

$$[i] = \frac{-g(R+S)\dfrac{d\theta}{dt}}{Q^2},$$

and

$$[I] = \frac{-gS\dfrac{d\theta}{dt}}{Q^2}.$$

Combining, we get the respective currents through seismometer and galvanometer when both move:

$$I = \frac{G(r+S)\dfrac{d\varphi}{dt}}{Q^2} - \frac{gS}{Q^2}\frac{d\theta}{dt}. \tag{20}$$

$$i = \frac{GS}{Q^2}\frac{d\varphi}{dt} - \frac{g(R+S)}{Q^2}\frac{d\theta}{dt}. \tag{21}$$

We may now write the equation of the *seismometer* when it is coupled to a galvanometer

$$K\frac{d^2\varphi}{dt^2} + D\frac{d\varphi}{dt} + U\varphi + hM\frac{d^2y}{dt^2} + GI = 0, \tag{22}$$

where:

K is moment of inertia,
D is a damping constant,
$\frac{U}{K} = n^2 = \left(\frac{2\pi}{T_0}\right)^2,$
h = distance of center of mass to hinge,
M = mass,

and

G = electrodynamic constant of seismometer.

We have written the sign of the term in d^2y/dt^2 as positive to conform with our earlier convention; for example, if earth motion north is considered positive, then a motion of the seismometer mass north relative to its support is also called positive. It is common procedure to write the sign of d^2y/dt^2 as negative. The equation states that the acceleration of the pendulum relative to its support is opposed by: (a) a damping force proportional to its velocity relative to its support, (b) its restoring force proportional to its displacement relative to its support, (c) a force proportional to the current in its coils; and that the acceleration of the pendulum is aided by a force proportional to the acceleration of the earth.

The equation of motion of galvanometer coil is

$$K_1 \frac{d^2\theta}{dt^2} + D_1 \frac{d\theta}{dt} + U_1\theta - gi = 0, \tag{23}$$

where

K_1 = moment of inertia of coil,
D_1 = damping constant,

and

$$\frac{U_1}{K_1} = n_1^2 = \frac{2\pi}{T_1^2},$$

where T_1 is the free period of the galvanometer coil and g is the electrodynamic constant of the galvanometer.

Note that there is no term in d^2y/dt^2. It is assumed that the galvanometer suspension passes through its center of mass, that is, the galvanometer must not be a seismometer

on its own account. The term in gi has a negative sign, for the current flowing does not oppose the motion of the galvanometer.

Substitute in 22 the value of I from 20 and set

$$2\kappa = \left(D + \frac{r+S}{Q^2}G^2\right)\frac{1}{K} = \frac{Q^2D + G^2(r+S)}{Q^2K}, \tag{24}$$

$$2\sigma_0\kappa = \frac{SGg}{Q^2K},$$

and

$$\frac{U}{K} = n^2.$$

Then 22 becomes

$$(p^2 + 2\kappa p + n^2)\varphi = 2\sigma_0\kappa p\theta - \frac{hM}{K}p^2y. \tag{25}$$

Putting

$$2\kappa_1 = \frac{Q^2D_1 + g^2(R+S)}{Q^2K_1}, \tag{26}$$

and

$$2\kappa_1\sigma_1 = \frac{SGg}{Q^2K_1},$$

and

$$\frac{U_1}{K_1} = n_1^2,$$

equation 23 becomes, using equation 21,

$$(p^2 + 2\kappa_1 p + n_1^2)\theta = 2\sigma_1\kappa_1 p\varphi. \tag{27}$$

If we use our solutions only for cases in which

$$\theta_0 = \dot\theta_0 = 0 \text{ and } \varphi_0 = \dot\varphi_0 = 0,$$

we may solve equation 27 for θ directly as

$$\theta = \frac{2\sigma_1\kappa_1 p\varphi}{p^2 + 2\kappa_1 p + n_1^2}$$

and substitute in 25 to get

$$[(p^2 + 2\kappa_1 p + n_1^2)(p^2 + 2\kappa p + n^2) - 4\sigma^2\kappa\kappa_1 p^2]\varphi = -\frac{hM}{K}(p^2 + 2\kappa_1 p + n_1^2)p^2y, \tag{28}$$

where $\sigma^2 = \sigma_0\sigma_1$.

Similarly, solving 25 for φ and substituting in 27, we get

$$[(p^2 + 2\kappa p + n^2)(p^2 + 2\kappa_1 p + n_1^2) - 4\sigma^2\kappa\kappa_1 p^2]\theta = -\frac{hM}{K} k_g p^3 y, \tag{29}$$

where

$$k_g = 2\sigma_0\kappa \frac{K}{K_1} = 2\sigma_1\kappa_1 = \frac{GgS}{Q^2K_1}$$

and is Galitzin's *transference factor*, one of the constants listed for Galitzin seismometers.

Equation 29 gives the relationship between earth displacement y and the galvanometer displacement θ which is observed.

Let us investigate the response of the galvanometer to simple harmonic earth motion. We consider the response after the transient has disappeared and therefore may consider it a problem with initial values of zero, since the steady state is independent of initial conditions. Take

$$\begin{aligned} y &= \sin \omega t \\ &= -ie^{i\omega t} \text{ (real part of)}, \\ py &= i\omega y, \end{aligned}$$

and

$$p^2 y = (i\omega)^2 y.$$

So we may replace the operator p acting on y by $i\omega$ for the simple harmonic motion. Now we know from experience that when the steady state is attained, θ will also be equal to a simple harmonic function of frequency ω. This function will differ from y by a magnification factor and a phase angle. We may therefore replace the operator p acting on θ by $i\omega$.

Whence 29 becomes

$$\begin{aligned} \theta &= \frac{-\dfrac{hM}{K} k_g p^3 y}{(p^2 + 2\kappa p + n^2)(p^2 + 2\kappa_1 p + n_1^2) - 4\sigma^2\kappa\kappa_1 p^2} \\ &= \frac{iF\omega^3}{(n^2 - \omega^2 + 2i\kappa\omega)(n_1^2 - \omega^2 + 2i\kappa_1\omega) + 4\sigma^2\kappa\kappa_1\omega^2}(-ie^{i\omega t}), \end{aligned}$$

where

$$F = \frac{hM}{K} k_g.$$

Put

$$We^{i\gamma} = \frac{iF\omega^3}{(n^2 - \omega^2 + 2i\kappa\omega)(n_1^2 - \omega^2 + 2i\kappa_1\omega) + 4\sigma^2\kappa\kappa_1 a^2}$$
$$= W(\cos\gamma + i\sin\gamma);$$

whence

$$\tan\gamma = \frac{\omega^4 - \omega^2[n^2 + n_1^2 + 4\kappa\kappa_1(1 - \sigma^2)] + n^2n_1^2}{2\omega(\kappa n_1^2 + \kappa_1 n^2) - 2\omega^3(\kappa + \kappa_1)}.$$

$$W = \frac{F\omega^3}{\sqrt{\{\omega^4 - \omega^2[n^2 + n_1^2 + 4\kappa\kappa_1(1 - \sigma^2)] + n^2n_1^2\}^2 + [2\omega(\kappa n_1^2 + \kappa_1 n^2) - 2\omega^3(\kappa + \kappa_1)]^2}}.$$

$$\begin{aligned}\theta &= -iWe^{i\gamma}e^{i\omega t}\\ &= -iWe^{i(\omega t + \gamma)}\\ &= -iW\{\cos(\omega t + \gamma) + i\sin(\omega t + \gamma)\}.\end{aligned}$$

The real part of θ is

$$\theta = W\sin(\omega t + \gamma),$$
$$x = L\theta = LW\sin(\omega t + \gamma),$$

where $L/2$ is the optical lever arm of the galvanometer recording system. Hence the dynamic magnification is

$$\vartheta = LW$$
$$= \frac{L\frac{hM}{K}k_g\omega^3}{\sqrt{\{\omega^4 - \omega^2[n^2 + n_1^2 + 4\kappa\kappa_1(1 - \sigma^2)] + n^2n_1^2\}^2 + [2\omega(\kappa_1 n^2 + \kappa n_1^2) - 2\omega^3(\kappa + \kappa_1)]^2}}.$$

In the particular case in which $n_1 = \kappa_1$ and $\sigma^2 = 0$ this equation degenerates into Galitzin's form.

$$\vartheta = \frac{Lk_g}{l\omega}\,\frac{1}{(1 + u_1^2)(1 + u^2)\sqrt{1 - \mu^2 f(u)}},$$

where

$$u_1 = \frac{n_1}{\omega}; \qquad u = \frac{n}{\omega}; \qquad \mu^2 = 1 - \frac{\kappa^2}{n^2},$$

and

$$f(u) = \left(\frac{2u}{1+u^2}\right)^2; \qquad l = \frac{K}{hM}.$$

The determination of the constants of an electromagnetic seismograph

The constants of a seismograph[3] using galvanometric recording are n_1, n, κ_1, κ, k_g, $\frac{hM}{K}$, σ, and L.

To compute n_1 and κ_1:

Remove the external circuit from the galvanometer. There will still be some damping on the galvanometer owing to air resistance and so forth. The damping in this condition we shall call κ_{10}. Allow the galvanometer to swing. Call the natural logarithm of the ratio of two successive amplitudes Λ_{10}. Then if $2\pi/n_{10}$ is the period in this partially damped condition,

$$\kappa_{10} = n_{10}\frac{\Lambda_{10}}{\pi},$$

and κ_{10} may be computed from the observed n_{10} and Λ_{10}. Also,

$$n_1 = \sqrt{n_{10}^2 + \kappa_{10}^2},$$

and n_1 may be computed. (The development of these equations is similar to that for the seismograph given on page 111.) From 26,

$$\kappa_{10} = \frac{D_1}{2K_1}.$$

To determine

$$\frac{g^2}{2K_1}$$

put a known low resistance shunt, S_1, across the galvanometer and again make the damping test, determining κ_{1s}, the value of κ_1 for this case. Now r is known, and S_1. and $R = \infty$. Hence from 26,

$$\kappa_{1s} = \frac{D_1}{2K_1} + \frac{1}{r+S_1}\frac{g^2}{2K_1},$$

and $g^2/2K_1$ may be determined. Now from 26, κ_1 may be computed for any value of S desired in the final arrangement of the system. $\kappa_1 = n_1$ considerably simplifies computations of earth motion.

For the seismometer, remove the external circuit and the damping magnets, if there are any. Allow the pendulum to swing. Its period will be $2\pi/n_0$. If there is still appreciable damping, it may be measured to get the values:

$$\kappa_0 = n_0 \frac{\Lambda_0}{\pi}$$

and

$$n = \sqrt{n_0^2 + \kappa_0^2}.$$

The n so computed may not be the effective n of the equations if the reintroduction of the magnets adds magnetic restoring force or tilts the seismograph. κ_0 is the residual mechanical damping which cannot be removed. Now put a known low resistance shunt, S_2, across the galvanometer and measure the corresponding κ_s.

$$\kappa_s = \kappa_0 + \frac{1}{R + S_2} \frac{G^2}{2K}$$

from equation 24, since $r = \infty$ in this experiment. Compute $G^2/2K$.

Remove the shunt, put in the damping magnets, and determine

$$\kappa' = \frac{D}{2K} = n' \frac{\Lambda'}{\pi},$$

the damping of the pendulum.

Now connect the system as it is to be used in service and compute from 24

$$\kappa = \kappa' + \frac{r + S}{Q^2} \frac{G^2}{2K}.$$

Now n_1, n, κ_1, κ are known.

It may be shown by combination of equations given above that

$$\sigma^2 = \frac{(\kappa_1 - \kappa_{10})(\kappa - \kappa')}{\kappa\kappa_1} \frac{S^2}{Q^2 + S^2},$$

so that σ may be computed. It is desirable to adjust S so that $\kappa_1 = n_1$.

The next problem is to determine k_g. A method of obtaining k_g is as follows: With the seismometer, galvanometer, and shunt connected as in service, give the seismometer a sudden known displacement. This may be done by moving it quickly by hand against a stop and holding it there. Then we may write

$$\varphi = \Phi_0 H(t),$$

where Φ_0 is the observable angular displacement given the pendulum. We have then

$$\frac{d^2\theta}{dt^2} + 2\kappa_1 \frac{d\theta}{dt} + n_1^2\theta = k_g\Phi_0 \frac{d}{dt} H(t)$$

for the galvanometer motion.

Integrate from $t = 0$ to $t = \eta$ where η is very small.

$$\left[\frac{d\theta}{dt}\right]_0^\eta + 2\kappa_1[\theta]_0^\eta = k_g\Phi_0,$$

$n_1^2 \int_0^\eta \theta\, dt$ being negligible, for θ can build to no appreciable value in so short a time. Now at $t = 0$, $d\theta/dt = \theta = 0$. At $t = \eta$, θ is negligible, so that

$$\left[\frac{d\theta}{dt}\right]_{t=\eta} = k_g\Phi_0.$$

We have therefore effectively given the galvanometer an initial velocity and may treat the problem as one in which the seismometer mass is not moved but the galvanometer is given an initial angular velocity of $k_g\Phi_0$.

The equation of galvanometer response when the seismometer is stationary is

$$(p^2 + 2\kappa_1 p + n_1^2)\theta = 0.$$

The solution is

$$\theta = p\,\frac{(p + 2\kappa_1)\theta_0 + \dot{\theta}_0}{p^2 + 2\kappa_1 p + n_1^2}$$

(compare equation 3).
Writing

$$p^2 + 2\kappa_1 p + n_1^2 = (p - \alpha)(p - \beta),$$
$$\alpha = -\kappa_1 - \sqrt{\kappa_1^2 - n_1^2}.$$
$$\beta = -\kappa_1 + \sqrt{\kappa_1^2 - n_1^2}.$$

The solution after equation 6 is

$$\theta = \frac{1}{\alpha - \beta}\,\{(\dot{\theta}_0 - \beta\theta_0)e^{\alpha t} - (\dot{\theta}_0 - \alpha\theta_0)e^{\beta t}\}.$$

which for our case, where $\theta_0 = 0$ and $\dot{\theta}_0 = k_g\Phi_0$, becomes

$$\theta = \frac{k_g\Phi_0 e^{-\kappa_1 t}}{\sqrt{n_1^2 - \kappa_1^2}}\sin\sqrt{n_1^2 - \kappa_1^2}\,t.$$

The first maximum θ_1 occurs at time t_1. If we observe θ_1 and t_1, by use of mirror and scale and stop watch, we may compute k_g from the above equation

$$k_g = \frac{\theta e^{\kappa_1 t_1}\sqrt{n_1^2 - \kappa_1^2}}{\Phi_0 \sin\sqrt{n_1^2 - \kappa_1^2}\,t_1}.$$

If $n_1 = \kappa_1$, that is, the galvanometer is critically damped, we have

$$\lim_{\sqrt{n_1^2 - \kappa_1^2} \to 0} \frac{\sin\sqrt{n_1^2 - \kappa_1^2}\,t}{\sqrt{n_1^2 - \kappa_1^2}} = t;$$
$$\theta = k_g\Phi_0 t e^{-\kappa_1 t}.$$

At θ_1, the first and only maximum $\kappa_1 t_1 = n_1 t_1 = 1$, that is

$$\frac{d}{dt}(te^{-\kappa_1 t}) = 0,$$

and

$$\theta_1 = 0.36788\Phi_0\frac{k_g}{n_1}.$$

If the free period of the galvanometer is very short, the accurate determination of t_1 is difficult. It might be obtained by recording on a fast drum. Again, if the seismograph is a vertical component instrument in which

the spring acts on a vertical line through the center of mass, it may be difficult to determine accurately the magnitude of the displacement as the mass is moved quickly against the stop. For such an instrument no rotation accompanies the displacement. When rotation is present, a mirror fastened at the axis rotates a reflected beam of light through twice the angle of rotation of the mass, and the angle of

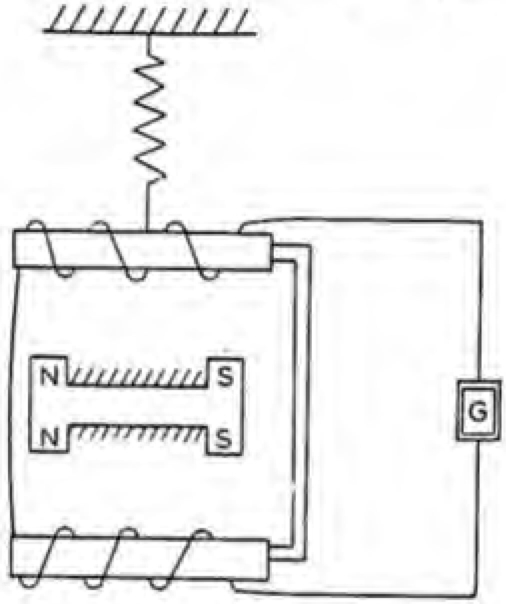

Fig. 33. Moving Armature Seismometer.

rotation may be determined as accurately as desired by increasing the optical lever arm of observation.

This and other experiments for obtaining k_g are well described in the papers of Rybner cited at the end of the chapter.

To determine $hM/K = 1/l$, the upset method or tilt method may be used as described above. L is one half the optical lever arm of the galvanometer.

The reader interested in the comparative response of seismometers of various constants would do well to read the paper of Coulomb and Grenet[4].

Benioff type

In some electromagnetic seismographs, particularly one developed by Benioff, an iron core or armature is used with

the coil. The core or the coil and core may be part of the pendulum mass with the magnets fastened to the frame, or vice versa. We shall discuss the particular case of a vertical seismograph with coils and cores fastened to the sprung mass and with spring force acting vertically through the center of mass[5] (see Figure 33).

The forces acting on the sprung mass in motion are (a) magnetic owing to induced magnetism in cores, (b) electromagnetic owing to induced current in circuit, (c) mechanical owing to damping, (d) mechanical owing to spring, (e) inertial, and (f) the gravitational force which is balanced by the initial spring tension and therefore need not enter the equation.

Let

R = reluctance of each air gap,
R_m = " " magnet,
R_a = " " each armature,
φ = flux through each air gap,
φ_m = " " magnet,
φ_a = " " each armature,
F = magnetomotive force of magnet.

At rest in either the upper or lower magnetic circuits

$$2\varphi R + \varphi_m R_m + \varphi_a R_a = F.$$

When the armatures move a small distance relative to the magnet,

$$2(\varphi + d\varphi)(R + dR) + (\varphi_m + d\varphi_m)R_m + (\varphi_a + d\varphi_a)R_a = F,$$

whence

$$2\varphi dR + 2Rd\varphi + 2d\varphi dR + R_m d\varphi_m + R_a d\varphi_a = 0.$$

Neglect R_m and R_a compared to R; then to first orders

$$d\varphi = -\frac{dR}{R}\varphi.$$

If l be the length of the air gap when the system is at rest, then

$$R = \frac{l}{A},$$

where A is the area of the pole pieces, and then

$$d\varphi = -\frac{dl}{l}\varphi.$$

Let z be the relative displacement of armatures and magnet, measured positive upward. Also measure forces tending to move sprung mass upward as positive.

The magnetic force on the lower armature is positive, while that on the upper armature is negative. At rest they balance. Consider the case of the armatures moving upward relative to the magnet; that is, z is positive. The change in force acting on the armatures at each gap will be the product of the field strength in the gap by $d\varphi$. That is, by moving the armatures up we have effectively weakened the induced poles in the upper armature by $d\varphi$ (and also the field strength in the gap). We have strengthened those for the lower armature by the same amount. Let F_1 be the resultant magnetic force. It is made up of two parts, an increase of the upward force and a decrease of the downward force.

$$F_1 = 2H_b d\varphi_b - 2H_u d\varphi_u,$$

where H_b and H_u are fields in lower and upper gaps and $d\varphi_b$, $d\varphi_u$ are changes in flux in lower and upper gaps. Whence

$$\begin{aligned} F_1 &= \frac{2}{A}\left(\varphi + \varphi\frac{z}{l}\right)\left(\varphi\frac{z}{l}\right) - \frac{2}{A}\left(\varphi - \varphi\frac{z}{l}\right)\left(-\varphi\frac{z}{l}\right) \\ &= \frac{4\varphi^2 z}{Al}. \end{aligned}$$

$$F_1 = \frac{\varphi_m{}^2}{A}\frac{z}{l}, \tag{30}$$

since $\varphi_m = 2\varphi$, the flux dividing equally between the two armatures in the rest position.

The flux through the armatures caused by a current, i, flowing through them is

$$\frac{Li}{n},$$

where L is that part of the inductance due to flux not lost by leakage and n is the number of turns in the coil. The force due to this flux is

$$F_2 = -2(H_b + H_u)\frac{Li}{n},$$

where i is taken as positive when the armatures move up relative to the magnet. Then the force is negative by Lenz's Law.

$$F_2 = -\frac{2\varphi_m}{nA}Li. \tag{31}$$

The damping force will be

$$F_3 = -D\frac{dz}{dt}, \tag{32}$$

where D is a constant; that is, it will oppose motion.

The effective spring force (its change over that part which just balances gravity at rest) is

$$F_4 = -Sz, \tag{33}$$

and the inertia force,

$$F_5 = -M\frac{d^2}{dt^2}(y + z), \tag{34}$$

where y is the displacement of the magnet and the earth.

Hence from 30 to 34 by addition

$$\frac{d^2z}{dt^2} + \frac{D}{M}\frac{dz}{dt} + \left(\frac{S}{M} - \frac{S'}{M}\right)z + \frac{2\varphi_m}{nAM}Li = -\frac{d^2y}{dt^2}, \tag{35}$$

where $S' = \frac{\varphi_m^2}{Al}$.

Thus S' reduces the effective spring constant, or in a horizontal component seismometer the gravitational restoring force. Note that magnetic particles in the damping vanes of seismometers have this same effect of modifying the coefficient of the displacement in the differential equation, that is, of modifying the effective free period of

the pendulum. Equation 35 is of the same general form as 22.

Types of suspension

Horizontal pendulums. The boom holding the mass is held nearly horizontal by wires attached to the mast. The pivot at which the boom is hinged to the mass may be a point in socket as in the Milne instruments or may be a Cardan hinge (flat steel spring) as in the large Bosch-Omori seismographs (see Figure 34a).

As shown in Figure 34b the horizontal pendulum is sometimes connected to the frame by two rigid members which are coupled by Cardan hinges to the frame. The lower hinge is a compression joint, and the upper a tension joint, although both may be arranged to hold the Cardan spring in tension. The Wilip-Galitzin seismometer uses this type of suspension.

The torsion suspension is shown in Figure 34c. A fine wire suspends a small cylinder eccentrically. The restoring force is partly torsional and partly gravitational. The Wood-Anderson seismograph is a torsion pendulum of this type.

The Zöllner suspension was employed by Galitzin and is illustrated in Figure 34d.

Inverted pendulums. In his horizontal seismograph Wiechert used an inverted pendulum which records both components of horizontal motion by use of a suitable system of recording pens. The pendulum is held in an almost vertical position by two small springs. In the smaller instruments the lower hinge is ball in socket, while in the larger it is built of crossed Cardan springs. Figure 34e illustrates this pendulum diagramatically.

Vertical motion pendulums. The mass may be suspended vertically from a coiled spring as in Figure 34f. The Benioff verticals are of this type.

The mass may be held by a horizontal flat spring as in some prospecting instruments (see Figure 34h).

To obtain longer free periods for vertical component pendulums, early seismometer builders were driven to suspensions of the type illustrated in Figure 34g. A boom

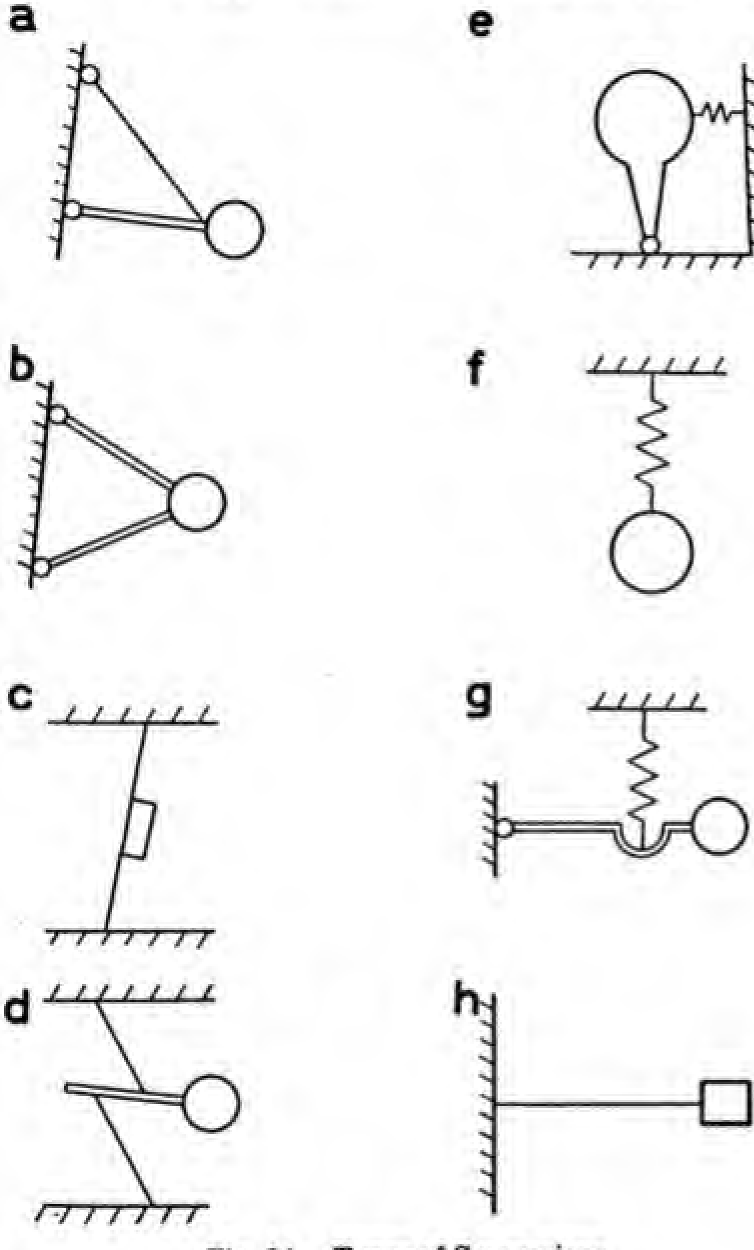

Fig. 34. Types of Suspensions.

which is horizontal in the rest position bears the mass at one end and is hinged at the other so that it may rotate in a vertical plane. A spring attached to the boom between mass and hinge establishes equilibrium when the boom is

horizontal. The hinge is normally composed of crossed Cardan springs. The spring is nearly vertical at the rest position in most of the vertical seismographs in use. Keeping the spring vertical, we may reduce the angle α, in Equation 17, by attaching the spring below the line joining the hinge and the center of mass. The length of the spring L when the mass is at rest may be left unchanged, its point of attachment to the frame being lowered also and the lever arm of the spring for the rest position left unaltered. From Equation 17 we see that this reduction of α will lengthen the free period. This lower attachment of the spring results in a decreased lever arm on downward motion and an increased lever arm on upward motion. Most vertical station seismographs, including Galitzin, Wilip-Galitzin and Wiechert types, are equipped with this general type of suspension. Some have additional springs for temperature compensation and to reduce lack of symmetry.

Nonpendulum seismometers

Benioff has developed an instrument called the "linear strain seismometer"[6]. Two piers are 20 meters apart. To the first of them is fastened one end of a pipe 20 meters long. This pipe extends horizontally to the second pier but is not fastened to it. To the free end of the pipe is attached one part of an electromagnetic transducer, while to the second pier is attached the other part of the transducer. Relative motion of the two piers results in relative motion of the free end of the pipe and the second pier. This motion causes an electromotive force to be induced in the transducer. The pipe is held horizontally by suitable supports which allow it to move longitudinally. The records of this instrument are very satisfactory.

Piezo electric crystals have been used with success in exploration seismographs, changes in pressure due to passage of elastic waves resulting in an electromotive force across the crystal.

The change in electrical resistance of a pile of carbon plates or carbon granules may also be used, the changes in pressure due to the passage of elastic waves changing the resistance of the pile. Some of the older geophones used this principle.

The passage of elastic waves causes changes in the resistivity of the ground which may be measured by suitable electrical apparatus.

Methods of recording

Mechanical systems. Many seismographs use lever systems connected to the pendulum, through which the record is written by a pen on smoked paper. The routine cost of operation of such recording is much less than the cost of recording on photographic paper. The objection to recording with a pen is that it introduces friction both at the contact of pen and paper and in the hinges of the lever system. If high magnification is desired, the mechanical advantage of this frictional force is very great and large masses must be used if the pendulum is to record. Some mechanically recording seismographs have masses as large as 20 metric tons.

Optical systems. Optically recording systems write by means of a beam of light reflected from a mirror on photographic paper. The mirror may be attached either to the pendulum or to a part of a galvanometer in circuit with the seismometer coil. In some seismographs the mirror is fastened rigidly to the mass and the magnification depends merely on the ratio of the mirror-record distance to the hinge-center of oscillation distance. The Wood-Anderson so records, although it uses a second mirror fixed to the seismometer base and hinged on a horizontal axis which doubles the magnification and renders easier the vertical adjustment of the light spot.

In other systems the end of the boom of the pendulum connected by a filament to a delicately balanced lever which

increases the magnification, although at the same time introducing some friction. The Milne-Shaw operates in this fashion.

Viscous coupling. Romberg and McComb have used viscous coupling in order to eliminate the recording of slow drifts of the pendulum. To the pendulum is attached a cup of oil into which dips a vane. The vane is hinged to the frame and connects to a mirror. When the pendulum moves rapidly, the vane moves with the oil, and the relative motion of pendulum and earth is recorded by a beam of light reflected by the mirror. It is only for very slow motions that the oil slowly flows around the vane and leaves no record. This eliminates the drifting of the zero line of the record.

Galvanometric systems. In the Galitzin and Wilip-Galitzin systems and in some Benioff models the coil is fastened to the boom of the pendulum and magnets to the frame of the supporting system. The coil is connected to the galvanometer, which records when relative motion of coil and magnet induces an electromotive force.

In some seismometers the coil is wound around a soft iron core, as discussed in the theory. Any one or two of the three transducer elements—coil, core, magnet—may be attached to the pendulum and the others to the frame. In early Benioffs the core only was attached to the pendulum.

The pendulum may be one plate of a condenser or attached to one, while the other plate is on the frame. Changes in capacity are then measured.

Damping

Oil damping. Oil damping is commonly used on large pendulums. The motion of vanes, attached to the pendulum, through oil in a cup on the pier produces the damping. The vanes should be so placed that it is the frictional drag of the oil on their surfaces and not the pushing out of the way of the oil that causes the resisting force. That is,

the vanes should move endways, not broadside, through the oil, because for ease in mathematical treatment it is desired that the resistance be proportional to the speed.

Air damping. Some of the older seismographs used air damping. In the Bosch-Omori instruments the pendulum was attached to a hinged rod which caused a large vane to move in a closed air chamber. In the Wiechert a piston was caused to move in a closed cylinder. In both systems there was difficulty with friction; in the one it was at the hinge, and in the other it was the difficulty in keeping clearance between piston and cylinder.

Electromagnetic damping. Electromagnetic damping is theoretically ideal. Either the mass itself (as in the Wood-Anderson) or plates attached to it (as in Galitzin, Milne-Shaw) are of copper and situated in a magnetic field, the magnets being attached to the frame of the seismograph. Relative motion of pendulum and frame induces eddy currents in the copper, which opposes motion with a force proportional to speed. The disadvantage of this type of damping is that in practical cases the copper is never free from magnetic impurities. The result is that the introduction of damping, after the "free period" has been measured, adds not only a damping force, but also a magnetic restoring force. This latter effectively alters the "free period" of the pendulum, which must then be computed indirectly. Again, addition of the heavy magnets after the determination of the period may tilt the base and alter the period in this way.

In the Wenner system the damping is obtained merely by an appropriate shunt in the circuit, there being no recourse to copper plates and eddy currents. Such a system has great advantage where the constants of seismometer and galvanometer allow sufficient damping to be so produced.

Recording drums and time

Most seismographs record on drums which rotate and translate at the same time. One of the major problems of

seismography is to obtain a constant rate of rotation for the drum. Driving the drum by a synchronous motor seems to give the best rate. In some stations (for example, Pasadena) the frequency of the current driving the motor is controlled by a vibrator, but in most stations the frequency as supplied by the ordinary power line is considered sufficiently uniform. In practice the rate of rotation of the record varies from 8 millimeters per minute (for example Milne-Shaw) to 60 millimeters per minute (for example, Wood-Anderson, Benioff), although of course these rates may be varied at individual stations. For drums of the larger rate, time may be read on the seismogram to tenths of a second, although the accuracy of time obtainable naturally depends on uniformity of drum rate, uniformity of the rate of the clock or chronometer marking the minutes, and the accuracy with which the clock error is determinable. A recording system recently developed by Benioff makes use of motion picture film. A perfected optical system writes a very fine line, and time may be measured to intervals much less than tenths of seconds.

It is necessary to have in a good station a pendulum clock or spring-driven chronometer of rate sufficiently uniform that it is possible to determine clock errors at any time with accuracy at least as great as the accuracy to which time can be measured on the record. Clock errors should be determined at least daily. These are determined by a number of stations by recording directly on the seismogram the radio time signals broadcast by one of the Naval Observatories. The correction may then be read with the same accuracy as other times on the record. Of course an auxiliary chronograph of higher rate may be used.

Accurate time is the *first* essential for a seismographic station. It is the stumbling block for the amateur seismologist. It is not extremely difficult for an amateur, clever with his hands, to build a sensitive seismometer. But such an instrument is almost useless for any scientific purpose if it does not record on a drum of very accurate

rate, on which time is marked by a good clock, the error of which is determined daily with accuracy.

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CHAPTER IX

Elastic Waves

General theory

The classical theory of elastic wave motion[1] is built upon the basis of the relationship between stress and strain as specified in the generalized form of Hooke's Law. *Stress* is the term used to denote the internal action and reaction upon each other of adjacent parts in the interior of an elastic solid. It is measured in terms of force per unit area. The force per unit area is sometimes referred to as traction or as intensity of stress, but more generally merely as stress.

To specify the stress across a given plane at a given point it is necessary to give the direction of the force, the magnitude of $\Delta F/\Delta A$ (where F is the force and A the area) as ΔA shrinks to zero, and the orientation of the area. A complete specification of stress at a point would involve the direction of the force and the magnitude of $\lim \Delta F/\Delta A$ across all the doubly infinite set of planes through the point. However, if the values of these quantities are given for three mutually perpendicular planes, the values for the others may be computed. It is convenient to express the force per unit area across each of these planes in the form of its three mutually perpendicular components, one measured normal to the plane and the other two tangentially to it. The three mutually perpendicular planes are taken as the coordinate planes.

Take for the three coordinates of a point the values x_i, where i may take the values 1, 2, 3. Consider the stress across three mutually perpendicular planes through the point, these planes being respectively parallel to the coordinate planes. Across each of these planes the force per unit

area may be resolved into three components. These may be expressed as p_{ij}, where j also may take the values 1, 2, 3. i represents the coordinate axis (x_1, x_2, or x_3) perpendicular to the plane in question. j represents the axis to which the component of force per unit area is parallel. The p used here is a force per unit area and not an operator as in the last chapter.

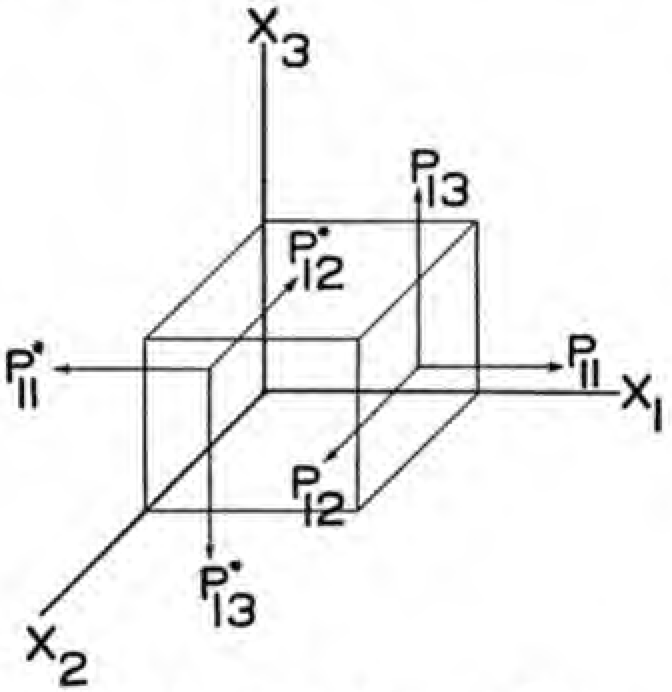

Fig. 35. Stresses on an Elementary Volume.

Consider an elementary parallelopiped in the elastic solid, one with its sides parallel to the coordinate planes. Forces acting on this little volume tending to make it move in the direction of the x_j axis may be of two kinds: (a) force due to stresses in the x_j direction on any of the faces, (b) body force in the x_j direction. Considering the stresses, we have across the faces perpendicular to x_i (see Figure 35) the stresses p_{ij} and p_{ij}^*. The net effect tending to produce translation in the x_j direction is the difference of these forces:

$$(p_{ij} - p_{ij}^*)dS_i = \frac{\partial p_{ij}}{\partial x_i} dx_i dS_i = \frac{\partial p_{ij}}{\partial x_i} d\tau,$$

to a sufficient approximation where dS_i is the area of each of the faces normal to x_i, and $d\tau$ is the volume. Across each pair of faces, specified by $i = 1, 2, 3$, there acts such a force. These, together with the body force per unit mass, F_j, in that direction, will produce an acceleration a_j.

$$a_j\rho d\tau = \left(\frac{\partial p_{ij}}{\partial x_i} + \frac{\partial p_{jj}}{\partial x_j} + \frac{\partial p_{mj}}{\partial x_m}\right) d\tau + F_j\rho d\tau,$$

where ρ is the density, or

$$\rho a_j = \frac{\partial p_{ij}}{\partial x_i} + \frac{\partial p_{jj}}{\partial x_j} + \frac{\partial p_{mj}}{\partial x_m} + \rho F_j. \tag{36}$$

There are three such equations, since x_j may represent any of the three axes.

Now the magnitudes of the tangential stresses are equal in pairs:

$$p_{ij} = p_{ji} \qquad (i \neq j).$$

If this were not so, the angular accelerations of the element of volume about axes through its center of mass would increase without limit as the dimensions of the element are reduced. Such may not be allowed in a continuous medium. (The angular acceleration acting on an elementary volume is equal to the force moment divided by the moment of inertia. Since the moment of inertia is of the magnitude of the squares of the elementary dimensions, so must be the resultant force moment, which is therefore negligibly small.)

Thus the nine components of stress reduce to six.

If a point with coordinates x_i, x_j, x_m is displaced elastically (strained) by an amount u_i in the direction of change in coordinate x_i, then an adjacent point of original coordinates $x_i + dx_i$, x_j, x_m will be displaced in the x_i direction by an amount $u_i + (\partial u_i/\partial x_i)dx_i$ to first orders. The length of the element of line between the two points after the strain is $(1 + e_{ii})dx_i$, where e_{ii} is called the *extension* of the line.

$$e_{ii} = \frac{\partial u_i}{\partial x_i} \text{ approximately.} \tag{37}$$

Such strain will in general be accompanied by some rotation, and the line after strain will make an angle α_{ii} with the x_i axis, where

$$\cos \alpha_{ii} = \frac{1 + \frac{\partial u_i}{\partial x_i}}{1 + e_{ii}},$$

where α_{ii} indicates the angle after strain between a line parallel to the x_i axis before strain and the x_i axes.

$$\cos \alpha_{ij} = \frac{\frac{\partial u_j}{\partial x_i}}{1 + e_{ii}} \qquad (i \neq j).$$

$$\cos \alpha_{im} = \frac{\frac{\partial u_m}{\partial x_i}}{1 + e_{ii}} \qquad (i \neq m).$$

The angle between two sides of an elementary volume which were parallel respectively to the x_i and x_j axes before strain becomes $\frac{\pi}{2} - \varphi$, where

$$\cos\left(\frac{\pi}{2} - \varphi\right) = \cos \alpha_{ii} \cos \alpha_{ji} + \cos \alpha_{ij} \cos \alpha_{jj} + \cos \alpha_{im} \cos \alpha_{jm}.$$

For small angular strains

$$\varphi = \frac{\left(1 + \frac{\partial u_i}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\left(1 + \frac{\partial u_j}{\partial x_j}\right) + \frac{\partial u_m}{\partial x_i}\frac{\partial u_m}{\partial x_j}}{(1 + e_{ii})(1 + e_{jj})}.$$

Substituting from equation 37, dropping small quantities of the second order, and setting for φ the symbol e_{ij}, we get:

$$e_{ij} = \frac{\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}}{1 + \frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_j}} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \text{ approximately.} \qquad (38)$$

The six values e_{ii} and e_{ij} are the six components of strain. Three are simple extensions, and three are small angles

representing the changes caused by strain in angles which before strain had sides parallel to the coordinate axes.

The generalized form of Hooke's Law states that each of the six components of stress is a linear function of all the components of strain.

$$p_{ii} = c_{ii}e_{ii} + c_{jj}e_{jj} + c_{mm}e_{mm} + c_{ij}e_{ij} + c_{jm}e_{jm} + c_{mi}e_{mi}. \tag{39}$$
$$p_{ij} = k_{ii}e_{ii} + k_{jj}e_{jj} + k_{mm}e_{mm} + k_{ij}e_{ij} + k_{jm}e_{jm} + k_{mi}e_{mi}. \tag{40}$$

where the c's and k's are constants. There are three equations of type 39 and three of type 40. Therefore there are 36 elastic constants.

For an isotropic medium the number of elastic constants degenerates to two.

In such a medium reversing the direction of any axis will not change the stress-strain relations, for the elastic constants are the same in all directions.

First: Reverse the direction of the x_i axis, leaving the x_j and x_m axes unchanged. The sign of p_{ii} must not change. It is the x_i component of the force per unit area acting on a surface normal to the x_i axis. The sign of that force has been reversed, but so has the sign of the area over which it acts; that is, the angle between the normal to that surface and the x_i axis has changed from 0 to π and its cosine from $+1$ to -1. From equation 37 it is seen that the signs of e_{ii}, e_{jj}, e_{mm} are not altered. Equation 38 indicates that the signs of e_{ij} and e_{mi} are altered. Therefore c_{ij} and c_{mi} must vanish for equation 39 to remain true when the direction of the x_i axis is reversed. e_{jm} is not changed in sign. However, a reversal of direction of either the x_j or x_m axes would change the sign of e_{jm} without affecting p_{ii}. So c_{jm} must vanish, and

$$p_{ii} = c_{ii}e_{ii} + c_{jj}e_{jj} + c_{mm}e_{mm}. \tag{41}$$

Now p_{ij} must change sign with the reversal of the x_i axis. It is the force per unit area in the x_j direction across a surface normal to the x_i axis. The sign of the force is not changed, but that of the area is. Therefore, in equation

40, k_{ii}, k_{jj}, k_{mm}, and k_{jm} must vanish to maintain the relation. Moreover, a reversal of direction of the x_m axis only would alter the sign of e_{mi} without changing that of p_{ij}, so k_{mi} must vanish, and 40 becomes

$$p_{ij} = k_{ij}e_{ij}. \tag{42}$$

Second: Interchange of any two axes must not alter the stress-strain relations. Consider the interchange of the axes x_j and x_m. Referring to equation 41, for this interchange to give p_{ii} in the same form it is necessary that the coefficients of e_{jj} and e_{mm} be identical. Therefore $c_{jj} = c_{mm} = c_2$. To preserve the form for interchange of x_i with x_j or x_m it is necessary that c_{ii} be the same for all three substitutions for i. We put $c_{ii} = c_1$, and 41 becomes

$$p_{ii} = c_1e_{ii} + c_2(e_{jj} + e_{mm}). \tag{43}$$

Again, in 42, to preserve the relation under interchange of axes k_{ij} must have the same value for all values of i and j. Therefore set $k_{ij} = c_3$, and 42 becomes

$$p_{ij} = c_3e_{ij}. \tag{44}$$

Third: Rotation of axes must not alter the form of equations 43 and 44. We rotate x_i and x_j axes through an angle ψ about the x_m axis to new axes which we call $x_i'x_j'$ (see Figure 36). Let $p_{ji}' = p_{ij}'$ denote the x_i' component of the force per unit area on the surface normal to the x_j' axis (equal in magnitude to the x_j' component on the surface normal to x_i'). Now p_{ji}' will be the compound of portions of p_{ii}, p_{jj}, p_{ij}, p_{ji} only, since no stresses normal to the x_i x_j plane will have components in it, nor will stresses across planes normal to the x_m axis have components across planes normal to the x_j' axis or to the x_i' axis. To get these components we must multiply the p_{ii} etc. first by the cosine of the angle the direction of the new force component makes with the old force component and second by the cosine of the angle the new surface makes with the old surface. Now the direction of p_{ii} (that of the x_i axis) makes an angle ψ

with x_i' (the direction of p_{ji}'). The surface over which p_{ii} acts is normal to x_i. The surface across which p_{ji}' acts is normal to x_j'. x_j' makes an angle $\frac{\pi}{2} + \psi$ with x_i, so the contribution of p_{ii} to p_{ji}' is $p_{ii} \cos \psi(- \sin \psi)$. Similarly the contribution of p_{jj} to p_{ji}' is $p_{jj} \sin \psi \cos \psi$. p_{ji} (the force per unit area in the x_i direction over a surface normal to

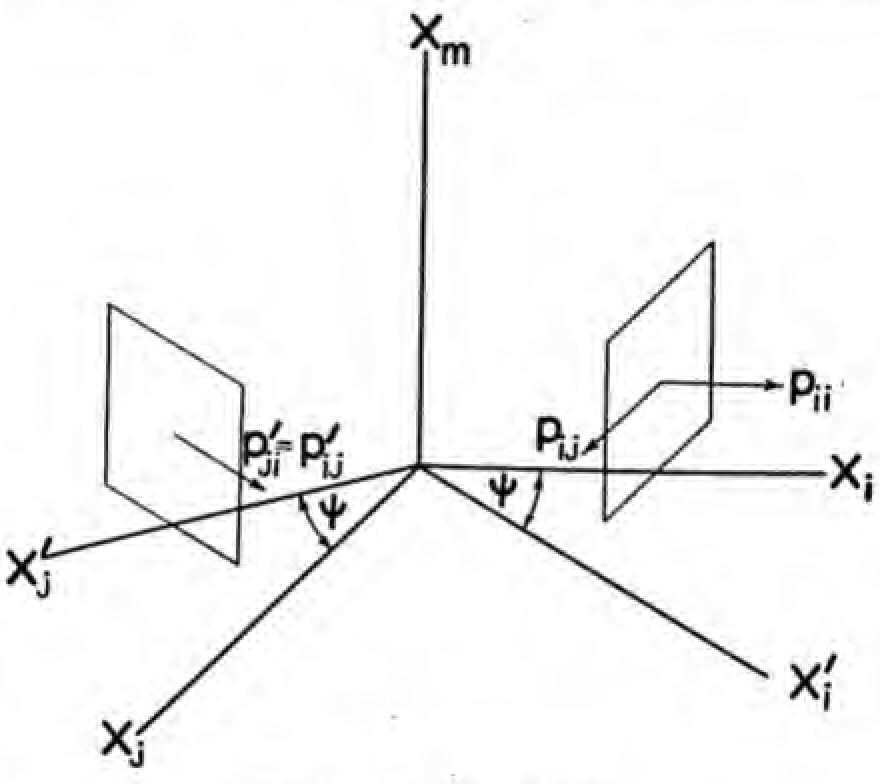

Fig. 36. Rotation of Axes.

x_j) is projected through $\cos \psi \cos \psi$ and p_{ij} through $\cos\left(\frac{\pi}{2} - \psi\right) \cos\left(\frac{\pi}{2} + \psi\right) = - \sin^2 \psi$.

Thus

$$p_{ji}' = p_{ij}' = -p_{ii} \cos \psi \sin \psi + p_{jj} \sin \psi \cos \psi + p_{ij}(\cos^2 \psi - \sin^2 \psi) \quad (45)$$

since $p_{ij} = p_{ji}$ in magnitude.

Substitute in 45 from 43 and 44:

$$p_{ij}' = \sin \psi \cos \psi (c_1 - c_2)(e_{jj} - e_{ii}) + (\cos^2 \psi - \sin^2 \psi) c_3 e_{ij}. \quad (46)$$

Now to obtain the strain component in the new system:

$$\frac{\partial u_i'}{\partial x_j'} + \frac{\partial u_j'}{\partial x_i'} = e_{ij}',$$

we note that

$$\frac{\partial u_i'}{\partial x_j'} = \left(\lambda_j \frac{\partial}{\partial x_i} + \mu_j \frac{\partial}{\partial x_j}\right)(\lambda_i u_i + \mu_i u_j),$$

where $\lambda_j \mu_j$ are the direction cosines of the x_j' axis with respect to the $x_i x_j$ axes, and $\lambda_i \mu_i$ are the direction cosines of the x_i' axis with respect to the same.

$$\frac{\partial u_i'}{\partial x_j'} = \lambda_j \lambda_i e_{ii} + \mu_j \mu_i e_{jj} + \lambda_j \mu_i \frac{\partial u_j}{\partial x_i} + \lambda_i \mu_j \frac{\partial u_i}{\partial x_j}.$$

Similarly,

$$\frac{\partial u_j'}{\partial x_i'} = \lambda_i \lambda_j e_{ii} + \mu_i \mu_j e_{jj} + \lambda_i \mu_j \frac{\partial u_j}{\partial x_i} + \lambda_j \mu_i \frac{\partial u_i}{\partial x_j}$$

$$e_{ij}' = 2\lambda_i \lambda_j e_{ii} + 2\mu_i \mu_j e_{jj} + (\lambda_i \mu_j + \lambda_j \mu_i) e_{ij}.$$

In our case

$$\lambda_i = \cos\psi; \qquad \lambda_j = -\sin\psi;$$
$$\mu_i = \sin\psi; \qquad \mu_j = \cos\psi.$$

So

$$e_{ij}' = \sin\psi\cos\psi(2e_{jj} - 2e_{ii}) + (\cos^2\psi - \sin^2\psi)e_{ij}. \tag{47}$$

Now if equation 44 is to be invariant for this rotation,

$$p_{ij}' = c_3 e_{ij}'. \tag{48}$$

Substituting 46 and 47 in 48, we see that 48 is invariant to this transformation only if

$$(c_1 - c_2)(e_{jj} - e_{ii}) = 2c_3(e_{jj} - e_{ii}).$$

Whence

$$c_1 = c_2 + 2c_3,$$

so there are only two elastic constants in an isotropic elastic solid. After Lamé we may set

$$c_3 = \mu,$$
$$c_2 = \lambda,$$
$$c_1 = \lambda + 2\mu,$$

and write equations 43 and 44 as

$$
\begin{aligned}
p_{ii} &= (\lambda + 2\mu)e_{ii} + \lambda(e_{jj} + e_{mm}),\\
p_{ii} &= \lambda\theta + 2\mu e_{ii},
\end{aligned}
\tag{49}
$$

where $\theta = e_{ii} + e_{jj} + e_{mm}$, the cubical dilatation.

$$p_{ij} = \mu e_{ij} \tag{50}$$

This μ is the coefficient of rigidity defined in Chapter I. Lamé's constant λ is related to the coefficient of incompressibility and the coefficient of rigidity by

$$\lambda = \kappa - \tfrac{4}{3}\mu.$$

Waves

Returning to equation 36, we may neglect body forces (F_j) in considering the vibrations due to elastic waves. We may also set

$$a_j = \frac{\partial^2 u_j}{\partial t^2}$$

approximately (the approximation being in the use of the *partial* derivative). We then have

$$\rho \frac{\partial^2 u_j}{\partial t^2} = \frac{\partial p_{ij}}{\partial x_i} + \frac{\partial p_{jj}}{\partial x_j} + \frac{\partial p_{mj}}{\partial x_m}. \tag{51}$$

Substituting 49 and 50 in 51 and rearranging, we get

$$\rho \frac{\partial^2 u_j}{\partial t^2} = (\lambda + \mu)\frac{\partial \theta}{\partial x_j} + \mu\nabla^2 u_j, \tag{52}$$

where ∇^2 indicates operation by

$$\frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial x_m^2}.$$

Operating on 52, with $\frac{\partial}{\partial x_j}$, we get

$$\rho \frac{\partial^2}{\partial t^2}\frac{\partial u_j}{\partial x_j} = (\lambda + \mu)\frac{\partial^2 \theta}{\partial x_j^2} + \mu\nabla^2 \frac{\partial u_j}{\partial x_j}.$$

Add the three equations of this type (subscripts $i\,j\,m$) and obtain

$$\frac{\partial^2\theta}{\partial t^2} = \frac{(\lambda + 2\mu)}{\rho}\nabla^2\theta. \tag{53}$$

This is the equation of propagation of cubical dilatation:

$$\theta = \frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_j} + \frac{\partial u_m}{\partial x_m}.$$

The speed of propagation is

$$v = \sqrt{\frac{\lambda + 2\mu}{\rho}}.$$

As such a disturbance passes through the elastic body (the earth in our case), the parts of the body experience change only in volume with no change in shape, only compression and rarefaction, with no shear.

Now operate on 52 with $\frac{\partial}{\partial x_i}$ to obtain

$$\rho\frac{\partial^2}{\partial t^2}\frac{\partial u_j}{\partial x_i} = (\lambda + \mu)\frac{\partial^2\theta}{\partial x_i \partial x_j} + \mu\nabla^2\frac{\partial u_j}{\partial x_i}.$$

Similarly,

$$\rho\frac{\partial^2}{\partial t^2}\frac{\partial u_i}{\partial x_j} = (\lambda + \mu)\frac{\partial^2\theta}{\partial x_j \partial x_i} + \mu\nabla^2\frac{\partial u_i}{\partial x_j}.$$

Subtracting

$$\rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right) = \mu\nabla^2\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right).$$

Call

$$\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j} = \omega_m.$$

$$\frac{\partial^2\omega_m}{\partial t^2} = \frac{\mu}{\rho}\nabla^2\omega_m. \tag{54}$$

The three equations of type 54 express the propagation of rotations with a speed

$$V = \sqrt{\frac{\mu}{\rho}}.$$

As such motion passes through an elastic body, it experiences only shearing strain with no change of volume.

The first preliminary (P) waves and the second preliminary (S) waves of seismology are waves of the types just described. P waves are irrotational, and S waves are equivoluminal.

Reflection and refraction

In general when an elastic wave of either the P or S type is incident on a boundary, there are reflected and refracted waves of both P and S types. An incident wave results in four transformed waves. There are of course particular cases in which some of these transformed waves are excluded (a) by the total reflection or refraction of one type or (b) by the nature of incident vibration excluding the existence of some of the transformed waves.

Snell's Law holds for elastic waves as for other types of waves.

$$\frac{\sin i_a}{\sin i_b} = \frac{v_a}{v_b},$$

where i_a is the angle of incidence and v_a the speed of the incident wave, where i_b is the angle of reflection or refraction and v_b the speed of the transformed wave; if $v_b > v_a$, there are values of i_a for which i_b does not exist. Again, when a P wave is incident normal to a discontinuity, there is no component of shear introduced at the boundary and so no transformed S waves. Or if the plane of vibration of S waves is parallel to the plane of the boundary, no dilatation component can be introduced at the boundary, since there is no motion normal to it.

We shall follow Zoeppritz [see Macelwane's *Geodynamics*[2]] in computing the relative amplitudes of incident, reflected, and refracted waves.

We shall deal with plane waves, and our problem will amount to a two-dimensional one. Let the x_2x_3 plane be the interface and the x_1x_3 plane the plane of incidence. Let a P wave be incident in the third quadrant, that is, the

domain of x_1 and x_3 negative (see Figure 37). Take $t = 0$ when the incident ray strikes the origin. Let the displacement of P incident along its path be

$$Ae^{ip\left(t - \frac{x_3 \sin \alpha + x_1 \cos \alpha}{v_1}\right)}$$

where v_1 is the speed of P waves in the medium of incidence.

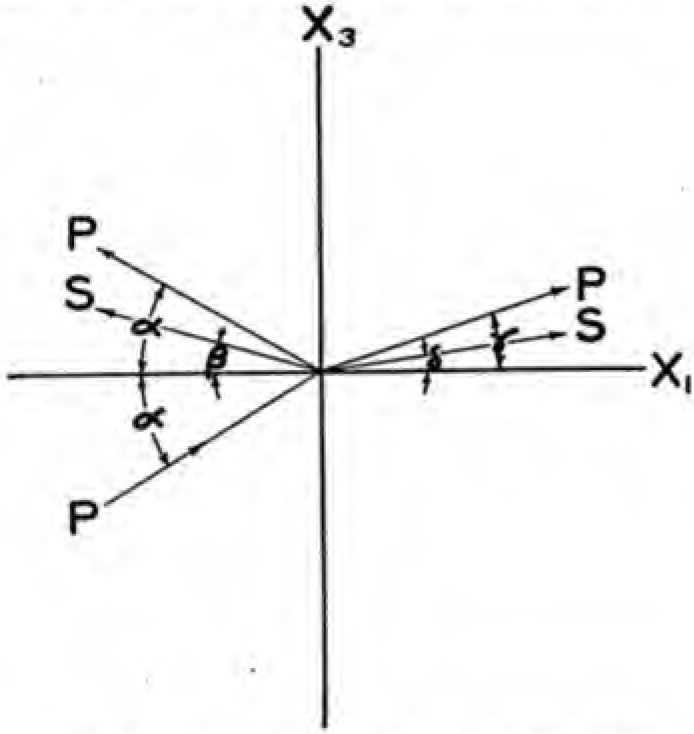

Fig. 37. Incident Reflected and Refracted Rays.

The equation of a wave front is

$$t - \frac{x_3 \sin \alpha + x_1 \cos \alpha}{v_1} = \text{constant} = 0$$

by our convention that $t = 0$ when $x_1 = x_3 = 0$.

$$t = \frac{x_3 \sin \alpha + x_1 \cos \alpha}{v_1},$$

that is, the time at which the incident wave front is at a distance $x_3 \sin \alpha + x_1 \cos \alpha$ from the origin is negative and its magnitude is obtained by dividing the distance it has to travel to get to the origin by its speed.

For the reflected and refracted P waves the displacements will be of form

$$A_1 e^{ip\left(t - \frac{x_2 \sin \alpha - x_1 \cos \alpha}{v_1}\right)},$$
$$A_2 e^{ip\left(t - \frac{x_2 \sin \gamma + x_1 \cos \gamma}{v_2}\right)},$$

v_2 being the speed of P waves in the medium of refraction. The angles are defined by the figure. They are angles of incidence, reflection, and refraction.

The reflected and refracted S displacements will be normal to their paths, will lie in the plane of incidence, and will be of form.

$$B_1 e^{ip\left(t - \frac{x_2 \sin \beta - x_1 \cos \beta}{V_1}\right)},$$
$$B_2 e^{ip\left(t - \frac{x_2 \sin \delta + x_1 \cos \delta}{V_2}\right)},$$

where V_1 and V_2 are the speeds of S waves in the media of incidence and refraction.

We shall assume that the components of displacement in the x_3 direction (u_3) of the reflected and refracted waves are in the same direction as in the incident wave. If in final computations we find an amplitude of one of the transformed waves to be negative, it will indicate that this assumption was unsound for this wave. For the transformed P waves the assumption amounts to that of no change in phase.

The displacement parallel to the x_1 axis (normal to the interface) owing to each of these five waves will be respectively.

$$\left.\begin{aligned}
&A e^{ip\left(t - \frac{x_2 \sin \alpha + x_1 \cos \alpha}{v_1}\right)} \cos \alpha, \\
&-A_1 e^{ip\left(t - \frac{x_2 \sin \alpha - x_1 \cos \alpha}{v_1}\right)} \cos \alpha, \\
&A_2 e^{ip\left(t - \frac{x_2 \sin \gamma + x_1 \cos \gamma}{v_2}\right)} \cos \gamma, \\
&B_1 e^{ip\left(t - \frac{x_2 \sin \beta - x_1 \cos \beta}{V_1}\right)} \sin \beta, \\
&-B_2 e^{ip\left(t - \frac{x_2 \sin \delta + x_1 \cos \delta}{V_2}\right)} \sin \delta.
\end{aligned}\right\} \qquad (55)$$

(The assumption of no change in phase at the boundary leads to the negative sign for reflected P waves and refracted S waves.) The P waves project through the cosine of the angle, while the S waves project through the cosine of $\pi/2$ minus the angle.

The components parallel to the x_3 axis are

$$\left.\begin{aligned}
&Ae^{ip\left(t - \frac{x_3 \sin\alpha + x_1 \cos\alpha}{v_1}\right)} \sin\alpha,\\
&A_1e^{ip\left(t - \frac{x_3 \sin\alpha - x_1 \cos\alpha}{v_1}\right)} \sin\alpha,\\
&A_2e^{ip\left(t - \frac{x_3 \sin\gamma + x_1 \cos\gamma}{v_2}\right)} \sin\gamma,\\
&B_1e^{ipt\left(t - \frac{x_3 \sin\beta - x_1 \cos\beta}{V_1}\right)} \cos\beta,\\
&B_2e^{ipt\left(t - \frac{x_3 \sin\delta + x_1 \cos\delta}{V_2}\right)} \cos\delta.
\end{aligned}\right\} \tag{56}$$

Now along the x_3 axis, the interface, there must be slipless contact between the two media, and therefore the sum of the displacements in the x_1 direction in medium 1 must equal the sum of those in medium 2 at the boundary. Whence, from 55.

$$A \cos\alpha - A_1 \cos\alpha + B_1 \sin\beta - A_2 \cos\gamma + B_2 \sin\delta = 0. \tag{57}$$

A similar requirement holds for the displacements in the x_3 direction, and

$$A \sin\alpha + A_1 \sin\alpha + B_1 \cos\beta - A_2 \sin\gamma - B_2 \cos\delta = 0. \tag{58}$$

Also the components of stress parallel to the x_1 axis must be continuous across the boundary. This relationship is obtained from equation 49 and is

$$\lambda_1\theta_1 + 2\mu_1(e_{11})_1 = \lambda_2\theta_2 + 2\mu_2(e_{11})_2. \tag{59}$$

For the plane problem

$$\theta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3},$$

and

$$e_{11} = \frac{\partial u_1}{\partial x_1},$$

where u_1 and u_3 are the displacements parallel to the x_1 and x_3 axes respectively. For u_1 we must set the sum of all the displacements parallel to x_1, and for u_3 the sum of those parallel to x_3 in each medium.

From 55 and 56 the values of these derivatives at $x_1 = 0$, the boundary, are

$$\left(\frac{\partial u_3}{\partial x_3}\right)_1 = \left(-A\,\frac{\sin^2\alpha}{v_1} - A_1\,\frac{\sin^2\alpha}{v_1} - B_1\,\frac{\sin\beta\cos\beta}{V_1}\right)e^{ip\xi},$$

where

$$\xi = t - \frac{x_3\sin\alpha}{v_1} = t - \frac{x_3\sin\beta}{V_1} = t - \frac{x_3\sin\gamma}{v_2} = t - \frac{x_3\sin\delta}{V_2},$$

and

$$\left(\frac{\partial u_1}{\partial x_1}\right)_1 = \left(-A\,\frac{\cos^2\alpha}{v_1} - A_1\,\frac{\cos^2\alpha}{v_1} + B_1\,\frac{\sin\beta\cos\beta}{V_1}\right)e^{ip\xi},$$

$$\lambda_1\theta_1 + 2\mu_1\left(\frac{\partial u_1}{\partial x_1}\right)_1 = \left\{-\frac{A + A_1}{v_1}(\lambda_1 + 2\mu_1\cos^2\alpha) + B_1\mu_1\,\frac{\sin 2\beta}{V_1}\right\}e^{ip\xi},$$

$$\begin{aligned}\lambda_1\theta_1 + 2\mu_1\left(\frac{\partial u_1}{\partial x_1}\right)_1 &= \{-\rho_1(A + A_1)v_1(1 - 2\sin^2\beta) + B_1\rho_1V_1\sin 2\beta\}e^{ip\xi}\\ &= (-A\rho_1v_1\cos 2\beta - A_1\rho_1v_1\cos 2\beta + B_1\rho_1V_1\sin 2\beta)e^{ip\xi}.\end{aligned}$$

Similarly,

$$\lambda_2\theta_2 + 2\mu_2\left(\frac{\partial u_1}{\partial x_1}\right)_2 = (-A_2\rho_2v_2\cos 2\delta + B_2\rho_2V_2\sin 2\delta)e^{ip\xi}.$$

Substituting in 59,

$$-A\cos 2\beta - A_1\cos 2\beta + B_1\frac{V_1}{v_1}\sin 2\beta + A_2\frac{\rho_2}{\rho_1}\frac{v_2}{v_1}\cos 2\delta - B_2\frac{\rho_2}{\rho_1}\frac{V_2}{v_1}\sin 2\delta = 0. \quad (60)$$

This equation expresses the continuity of normal stress across the interface.

The fourth boundary condition is that the tangential stresses must be continuous across the boundary, that

$$\mu_1\left\{\left(\frac{\partial u_1}{\partial x_3}\right)_1 + \left(\frac{\partial u_3}{\partial x_1}\right)_1\right\} = \mu_2\left\{\left(\frac{\partial u_1}{\partial x_3}\right)_2 + \left(\frac{\partial u_3}{\partial x_1}\right)_2\right\}. \quad (61)$$

Substituting the sum of the first, second, and fourth terms of 55 for u_1 in the left-hand member and the sum of the third and fifth for u_1 in the right-hand member, and similarly for u_2 from 56, we may evaluate the terms in 61, which becomes

$$-A \sin 2\alpha + A_1 \sin 2\alpha + B_1 \frac{v_1}{V_1} \cos 2\beta + A_2 \frac{v_1}{v_2}\frac{\rho_2}{\rho_1}\frac{V_2^2}{V_1^2} \sin 2\gamma + B_2 v_1 \frac{\rho_2}{\rho_1}\frac{V_2}{V_1^2}\frac{\rho_2}{\rho_1} \cos 2\delta = 0. \quad (62)$$

This equation expresses the continuity of tangential stress across the interface.

Now equations 57, 58, 60, and 62 may be solved for A_1/A, B_1/A, A_2/A, and B_2/A, giving the relative amplitudes of the reflected and refracted waves in terms of the amplitude of the incident P wave, for any angle of incidence, α. We have of course from Snell's Law that

$$\frac{\sin \alpha}{v_1} = \frac{\sin \beta}{V_1} = \frac{\sin \gamma}{v_2} = \frac{\sin \delta}{V_2}.$$

Apparent angle of incidence. Let $\bar{u}_3$ represent the component of the displacement of the free surface of the earth along itself. Then $x_1 = 0$, and we put $x_3 = 0$ for brevity. From 56

$$\bar{u}_3 = e^{ipt}\{(A + A_1) \sin \alpha + B_1 \cos \beta\}.$$

From 60, the surface being free, $A_2 = B_2 = 0$ very approximately,

$$A + A_1 = \frac{V_1}{v_1}\frac{B_1 \sin 2\beta}{\cos 2\beta} = \frac{\sin \beta}{\sin \alpha}\frac{\sin 2\beta}{\cos 2\beta} B_1.$$

Substituting, we get

$$\bar{u}_3 = e^{ipt}B_1\left\{\frac{\sin\beta\sin 2\beta}{\cos 2\beta} + \cos\beta\right\}$$

$$= e^{ipt}\frac{B_1}{\cos 2\beta}\{\sin\beta\sin 2\beta + \cos\beta\cos 2\beta\}.$$

$$\bar{u}_3 = e^{ipt}B_1\frac{\cos\beta}{\cos 2\beta}.$$

Similarly from 55 the surface displacement in the direction normal to it, $\bar{u}_1$, is

$$\bar{u}_1 = e^{ipt}\{(A - A_1)\cos\alpha + B_1\sin\beta\}.$$

and from 62 for a free surface

$$A - A_1 = B_1\frac{\sin\alpha}{\sin\beta}\frac{\cos 2\beta}{\sin 2\alpha} = B_1\frac{\cos 2\beta}{2\sin\beta\cos\alpha}.$$

Substituting, we get

$$\bar{u}_1 = e^{ipt}\frac{B_1}{2\sin\beta}\{\cos 2\beta + 2\sin^2\beta\}$$

$$= e^{ipt}\frac{B_1}{2\sin\beta}.$$

Now the apparent angle of incidence, $\bar{\alpha}$, is computed by

$$\tan\bar{\alpha} = \frac{\bar{u}_3}{\bar{u}_1}.$$

Therefore

$$\tan\bar{\alpha} = \frac{2\sin\beta\cos\beta}{\cos 2\beta} = \tan 2\beta.$$

So

$$\bar{\alpha} = 2\beta$$

and

$$\sin\alpha = \frac{v_1}{V_1}\sin\beta = \frac{v_1}{V_1}\sqrt{\frac{1-\cos\bar{\alpha}}{2}}. \tag{63}$$

In any study of angles of incidence based on amplitude ratios the value obtained from earth amplitudes must be converted to the true angle by the formula (see Figure 38).

Transverse waves incident. Similar equations may be set up for incident transverse waves, the two cases of waves vibrating in a vertical plane and in a horizontal plane being treated separately. For the second case the problem is

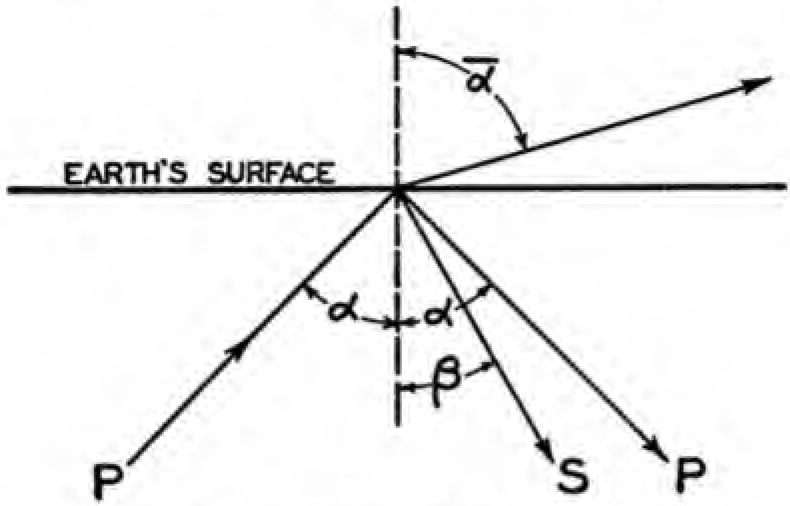

Fig. 38. Apparent Angle of Emergence.

greatly simplified, since no longitudinal waves can be set up when the transverse vibrations are normal to the plane of incidence.

Surface waves

Rayleigh waves. It was Lord Rayleigh[3] who first gave the theory for plane waves traveling along the boundary of a semi-infinite elastic solid. Let the x_2x_3 plane be the free surface and the positive x_1 axis extend down into the interior.

In equation 52 put

$$u_j = e^{ipt}f(x_1, x_2, x_3) \tag{64}$$

and get

$$(\nabla^2 + k^2)u_j = \left(1 - \frac{k^2}{h^2}\right)\frac{\partial\theta}{\partial x_j}, \tag{65}$$

where

$$k^2 = \frac{\rho p^2}{\mu} \quad \text{and} \quad h^2 = \frac{\rho p^2}{\lambda + 2\mu}. \tag{65a}$$

The particular solution of equation 65,

$$u_j = -\frac{1}{h^2}\frac{\partial\theta}{\partial x_j},$$

takes care of the dilational component of the Rayleigh wave.

We assume that θ is harmonic and is of the form

$$\theta = e^{-q_1x_1 + i(f_2x_2 + f_3x_3 + pt)},$$

that is, the wave travels along the x_2x_3 plane and dies off with depth. Hence we may write for the particular solutions

$$\left.\begin{aligned}(u_1)_p &= \frac{q_1}{h^2}\,e^{-q_1x_1}e^{i(f_2x_2+f_3x_3+pt)},\\ (u_2)_p &= -\frac{if_2}{h^2}\,e^{-q_1x_1}e^{i(f_2x_2+f_3x_3+pt)},\\ (u_3)_p &= -\frac{if_3}{h^2}\,e^{-q_1x_1}e^{i(f_2x_2+f_3x_3+pt)}.\end{aligned}\right\} \tag{66}$$

θ must satisfy (from 53 and 64)

$$\nabla^2\theta = -h^2\theta.$$

Hence

$$q_1^2 = f_2^2 + f_3^2 - h^2 \tag{67}$$

The complementary solution of 65 solves

$$(\nabla^2 + k^2)u_j = 0, \tag{68}$$

and therefore cares for the shear component of Rayleigh waves.

Taking

$$(u_j)_c = A_je^{-q_2x_1 + i(f_2x_2+f_3x_3+pt)}, \tag{69}$$

that is, components of same wave length and frequency as those in 66 but a different law of decrement with depth, and substituting in 68, we get

$$q_2^2 = f_2^2 + f_3^2 - k^2. \tag{70}$$

Also, since solution 69 must satisfy $\theta = 0$,

$$i(f_2A_2 + f_3A_3) - q_2A_1 = 0. \tag{71}$$

The complete solutions are the sums of equations 66 and 69.

When boundary conditions are introduced, the tangential and normal stresses must vanish at the x_2x_3 plane, since it is a free surface. These stresses are given by equations 49 and 50. Substituting the complete solutions for u_j in 50 gives, at $x_1 = 0$, stress vanishing,

$$2\frac{if_2q_1}{h^2} + if_2A_1 = q_2A_2, \tag{72}$$

and

$$2\frac{if_3q_1}{h^2} + if_3A_1 = q_2A_3. \tag{73}$$

From 71, 72, 73

$$A_1(f_2^2 + f_3^2 + q_2^2)h^2 + 2q_1(f_2^2 + f_3^2) = 0 \tag{74}$$

Substituting the complete solutions for u_j in 49 gives, for vanishing stress at $x_1 = 0$,

$$k^2 - 2h^2 - 2q_1^2 - 2q_2h^2A_1 = 0,$$

which, from 67, becomes

$$A_1 = \frac{k^2 - 2(f_2^2 + f_3^2)}{2q_2h^2}. \tag{75}$$

Eliminating A_1 by combining 74 and 75, q_2 from 70, and q_1 from 67, we get

$$\{k^2 - 2(f_2^2 + f_3^2)\}^2\{k^2 - 2(f_2^2 + f_3^2)\}^2 = 16(f_2^2 + f_3^2)^2 (f_2^2 + f_3^2 - h^2)(f_2^2 + f_3^2 - k^2). \tag{76}$$

Now the speed of Rayleigh waves is

$$V_r = \frac{p}{\sqrt{f_2^2 + f_3^2}},$$

the speed of P waves is

$$v = \frac{p}{h} = \frac{1}{a},$$

and the speed of S waves is

$$V = \frac{p}{k} = \frac{1}{b}, \text{ where } a \text{ and } b \text{ are here defined.}$$

Hence equation 76 becomes

$$b^6V_r^6 - 8b^4V_r^4 + (24b^2 - 16a^2)V_r^2 - 16\left(1 - \frac{a^2}{b^2}\right) = 0.$$

If Poisson's ratio is $\frac{1}{4}$,

$$a^2 = \tfrac{1}{3}b^2,$$

and there are three roots for $b^2V_r^2$, of which only that one less than unity is relevant, since otherwise q_2^2 in equation 70 would become negative. This root is 0.8453, whence

$$V_r = 0.9194\sqrt{\frac{\mu}{\rho}}.$$

Therefore Rayleigh waves travel along the surface of a semi-infinite elastic solid with a speed of about nine tenths that of shear waves in the interior.

We may add solutions as given by equations 66 and 69. A_1, A_2, A_3 may be eliminated by use of equations 71, 72, 73, leaving

$$\frac{h^2u_2}{f_2} = i\left\{-e^{-q_1x_1} + \frac{2q_1q_2}{f_2^2 + f_3^2 + q_2^2}\,e^{-q_2x_1}\right\}e^{i(f_2x_2+f_3x_3+pt)},$$
$$\frac{h^2u_3}{f_3} = i\left\{e^{-q_1x_1} + \frac{2q_1q_2}{f_2^2 + f_3^2 + q_2^2}\,e^{-q_2x_1}\right\}e^{i(f_2x_2+f_3x_3+pt)},$$
$$\frac{h^2u_1}{q_1} = \left\{e^{-q_1x_1} - \frac{2(f_2^2 + f_3^2)}{f_2^2 + f_3^2 + q_2^2}\,e^{-q_2x_1}\right\}e^{i(f_2x_2+f_3x_3+pt)}.$$

If the wave travels in the direction of u_2, f_3 and therefore u_3 vanish. If, moreover, Poisson's ratio is $\frac{1}{4}$, the real part of the equations of displacement becomes

$$\frac{h^2u_2}{f_2} = (e^{-q_1x_1} - 0.5773e^{-q_2x_1})\sin(f_2x_2 + pt),$$
$$\frac{h^2u_1}{f_2} = (0.8475e^{-q_1x_1} - 1.4679e^{-q_2x_1})\cos(f_2x_2 + pt).$$

At the surface $x_1 = 0$, and the equation of the path of a particle becomes

$$\frac{u_2^2}{\frac{f_2^2}{h^4}(0.4227)^2} + \frac{u_1^2}{\frac{f_2^2}{h^4}(0.6204)^2} = 1.$$

This is the standard form for an ellipse. The major axis is vertical (direction of u_1). The plane of vibration is the plane of propagation. The motion is retrograde. It dies off with depth owing to the exponential factors in x_1. The horizontal component of motion becomes zero at a depth of 0.192 of a wave length, and below this has a sign opposite to that at the surface. The vertical motion does not so change sign.

Rayleigh's theory has been modified for an earth in which the speed of waves increases with depth. The literature is large[4]. Love, Stoneley, Sezawa, Lee, Jeffreys have considered the effect of surface layering. Stoneley has tried an incompressible earth with the rigidity a linear function of depth. Pekeris expresses the speed of Rayleigh waves as a power series in the wave length and then treats the problems of several media. These introduce dispersion and explain the fact that waves similar to Rayleigh waves observed on seismograms show a vertical amplitude of about the same magnitude as the horizontal.

Love waves. Early in seismography there was observed at the beginning of the surface waves a motion transverse to the path joining epicenter and station and lacking a vertical component.

To explain such waves, Love postulated a layer of one set of constants overlying a medium of other constants. Since the motion is only in one direction, the theory is fairly simple.

The equation of motion is

$$(\nabla^2 + k^2)u_3 = 0. \tag{77}$$

Let the waves propagate in the x_2 direction with vibration in the x_3 direction, where $x_1 = 0$ is the lower boundary of the layer and $x_1 = H$ its free surface. For the layer assume

$$u_3 = (A \cos sx_1 + B \sin sx_1) \cos (fx_2 + pt + \epsilon), \tag{78}$$

where u_3 is the displacement.

Substituting 78 in 77, we get

$$s^2 = k^2 - f^2.$$

If we use primed letters for the lower medium and assume

$$u_3' = Ce^{s'x_1} \cos (fx_2 + pt + \epsilon),$$

we get

$$s'^2 = f^2 - k'^2. \tag{78a}$$

To have a surface wave u' must attenuate with depth, s' must be real (and positive), and therefore $f^2 > k'^2$. It follows that the speed of Love waves is less than that of shear waves in the lower medium.

Putting in the boundary conditions:

(a) Tangential displacements continuous across the interface give

$$A = C.$$

(b) Tangential stress continuous across the interface gives

$$\mu \frac{\partial u_3}{\partial x_1} = \mu' \frac{\partial u_3'}{\partial x_1} \qquad \text{at} \qquad x_1 = 0,$$

whence

$$\mu s B = \mu' s' C = \mu' s' A.$$

(c) Stress vanishes at $x_1 = H$ (free surface); and we get

$$\mu \frac{\partial u_3}{\partial x_1} = 0 \qquad \text{at} \qquad x_1 = H,$$

whence

$$\tan sH = \frac{B}{A} = \frac{\mu' s'}{\mu s}.$$

Expanding s' from 78a and 65a,

$$\tan sH = \frac{\mu'}{\mu} \left\{ \frac{f^2}{s^2} \left(1 - \frac{V^2}{V'^2} \right) - \frac{V^2}{V'^2} \right\}^{\frac{1}{2}}, \tag{79}$$

where V and V' are the speeds of shear waves in the layer and lower medium.

Since $f = 2\pi/\Lambda$ where Λ is the wave length of the Love wave and

$$k = \frac{p^2}{V^2},$$

equation 79 relates the thickness of the layer, the rigidities of layer and the medium, and the speeds of shear waves in them with the period, and wave length of the Love wave. Given the constants of layer and medium, the speed of Love wave of given period may be computed. The dispersion curve so obtained may be compared with the observed. It is to be remembered that the wave length and period so involved give as quotient the wave velocity (not group velocity). On computing the velocity of a seismic surface wave which suffers dispersion by dividing the distance traveled by the time taken, one obtains the group velocity if the wave measured is not the first wave of the series; if it is the first crest, there is some doubt. Study of the Love waves on the records of large earthquakes suggests that the phase of the first crest in a given direction from the epicenter does not change with epicentral distance and therefore may be observed as traveling with wave velocity.

Various investigators have shown that gradual increase of speed with depth will also permit the existence of Love waves[5]. Meissner assumed $v = \sqrt{1 + \delta z}\, v_0$, where δ is a constant and z the depth. Sezawa has taken $\mu = c(d + z)$, where c and d are constants. Aichi and Wilson have each postulated

$$\rho = \rho_0 e^{\beta z}, \qquad \mu = \mu_0 e^{\alpha z}, \ \alpha \text{ and } \beta \text{ being constants.}$$

The effect of several layers near the earth's surface has been computed by Stoneley.

Observations of surface waves. The surface waves are observed on seismograms generally as longer period, larger amplitude waves which follow the preliminary tremors (P, S, and their reflections). On long-period seismographs (say 8 seconds and over) these waves dominate the record for all but deep-focus earthquakes. They carry larger earth amplitudes than the preliminary waves. On very short period seismographs (period of half second, say) the surface waves are not conspicuous because the magnifica-

tion of such an instrument for long periods is very small. There appear to be three kinds of surface waves, whereas the theory accounts for only two. (See Figure 6.) All three types are not always noticeable on a seismogram. The first group is of the nature of vibration of the theoretical Love wave; that is, the vibration is transverse to the path joining epicenter and station and has no vertical component. These may be called G waves. Their periods are long (say 25 seconds to 3 minutes), and they often carry considerable earth amplitudes. For instance, the G waves of the South Atlantic earthquake of August 28, 1933, as recorded at Harvard, Massachusetts ($\Delta = 109°$), carried an amplitude of two millimeters. They are followed by a group of waves of shorter period although still long compared to the preliminary waves, often irregular in shape, and with a vertical component. These are called L waves. The earliest seismographical observers apparently missed the G waves. They called the second group L for "longus" for the waves are relatively long. Since tables of the travel times of this L group were in circulation before the long Love wave, G, was recognized, it has not been practicable to shift the symbol L to designate the long Love wave, G, where it logically belongs. There is thus some confusion in the use of L, some meaning by it the appearance of the Love wave, and some the appearance of the second surface group tabulated as L by Macelwane and others. We shall use it in the latter sense. Usually L merges gradually into shorter sinusoidal waves conspicuous on the records and of largest trace amplitude. These are designated as M (for "maximus"). Occasionally this group begins sharply as a new type of motion. There is some confusion in station bulletins regarding the use of M. To some it designates the time of arrival of the largest amplitude on the record; to others it means the beginning of the sinusoidal group carrying large amplitudes. The sinusoidal motion persists for a long time after the maximum is passed in the recording of a large distant earthquake. These "tail" waves are

called the *coda*. The *M* group and the coda frequently approximate Rayleigh waves in their motion[6]. Since *L* waves precede them closely, the beginning of *M* must be underlain by the end of *L*. The only group of which we could expect purity would be *P*, which is first and uncontaminated, and the coda which persists after the other waves are gone. All types of surface waves may be recorded late in the record of a large shock when they reach the station by the long path around the earth from the epicenter—the path which traverses the anticenter (180° from the epicenter). If the earthquake is large enough, the surface waves which have traveled the angular distance $(360° + \Delta)$ and even $(720° - \Delta)$ may be observed, Δ measuring the epicentral distance of the station. All these waves are called *returns* of the surface waves.

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CHAPTER X

Paths of Waves and Travel Time Curves

Introduction

One of the earliest activities of instrumental seismologists was the plotting of *travel time curves.* In such a curve the time of travel of the first motion in a given group of waves is plotted against the epicentral distance around the earth's surface; or perhaps merely the arrival time is plotted against the epicentral distance. The difference in the two cases is that the first requires a knowledge of the time when the wave started. Some writers call the second kind of curve a *time distance curve. Hodograph* was an early term applied to these curves.

For the surface waves, although there is a wide scattering of points on the graph, a straight line is a good approximation of the travel time curve. This result is the prime evidence that they are surface waves. The scattering is to be expected, for the earth's surface structure is complex. In particular, surface waves of a given period travel faster under oceans than under continents. Long surface waves travel faster than short ones, owing to their deeper penetration.

The travel time curves of P and S (and their reflections at the earth's surface) are concave downward, that is, the apparent surface speed increases with the epicentral distance. This indicates that their paths do not follow the earth's surface (see Chapter I). The speed of seismic waves increases with depth in the earth. Figure 7 shows the form of wave fronts and rays resulting from an increase of speed with depth.

General theory

A major problem of seismology is: Given a travel time curve, to find the speed at any depth.

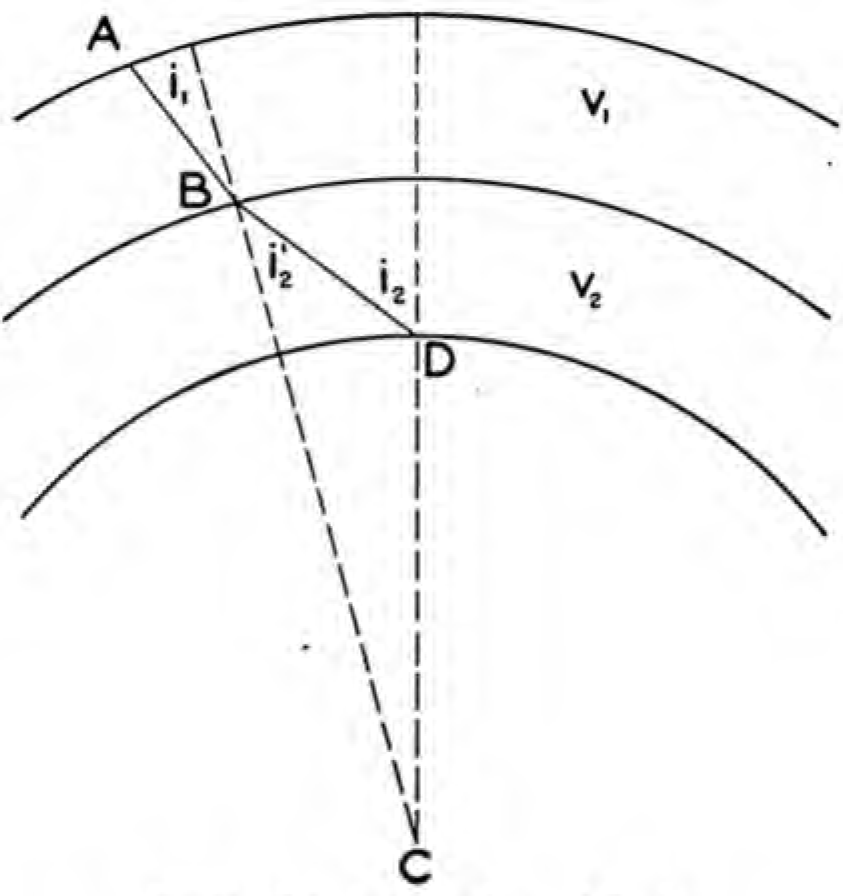

Fig. 39. Seismic Ray in Layered Earth.

Let Figure 39 represent a section of the earth with C as center. ABD is a portion of a ray passing through two shells in each of which the wave speed is constant (v_1 and v_2 respectively). By Snell's Law

$$\frac{\sin i_1}{\sin i_2'} = \frac{v_1}{v_2}.$$

In Δ BDC by the law of sines

$$\frac{\sin i_2}{\sin i_2'} = \frac{BC}{DC} = \frac{r_1}{r_2},$$

where $BC = r_1$; $DC = r_2$.
Hence

$$\frac{r_1 \sin i_1}{v_1} = \frac{r_2 \sin i_2}{v_2},$$

This relationship may be carried through all layers, and the layers may be shrunk as small as desired. We therefore have the relationship that along any one ray

$$\frac{r \sin i}{v} = \text{constant} = p.$$

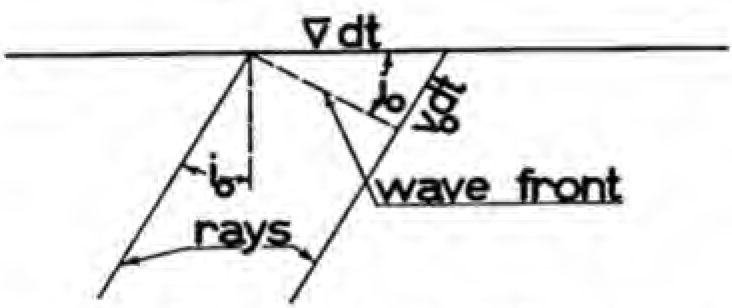

Fig. 40. Illustrating Apparent Surface Velocity.

At the earth's surface

$$\frac{r_0 \sin i_0}{v_0} = p.$$

But from Figure 40

$$\sin i_0 = \frac{v_0}{\bar{v}},$$

where $\bar{v}$ is the apparent surface speed of the wave.
Hence

$$p = \frac{r_0}{\bar{v}} = \frac{r_0}{\left(\frac{dx}{dt}\right)_{r=r_0}} = \frac{dt}{\frac{dx}{r_0}} = \left(\frac{dt}{d\theta}\right)_{r=r_0} = \frac{\partial t}{\partial \theta}. \tag{80}$$

x is arcual distance along the earth's surface and θ is the angle subtended by the ray at the earth's center.

To set up the equations of the travel time along a ray, consider Figure 41. AB and $A'B'$ are successive wave fronts. CP is a ray, P having coordinates xyz. P' is a

neighboring point of coordinates $x + dx$, $y + dy$, $z + dz$. It lies on ray $C'EP'$. The difference in time of arrival of the wave at P' and P is

$$dt = \frac{EP'}{v},$$

where v is the speed of the wave.

If l, m, n, are the direction cosines of element of ray EP',

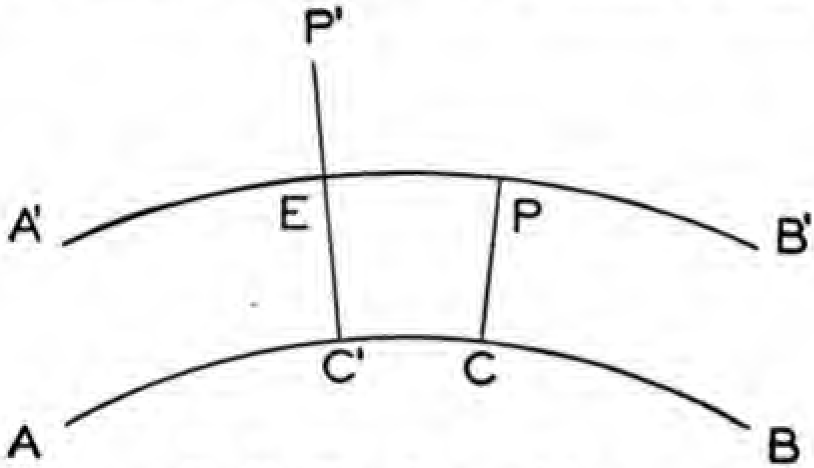

Fig. 41. Wave Fronts and Rays.

$$dt = \frac{EP'}{v} = \frac{ldx + mdy + ndz}{v};$$

$$\frac{\partial t}{\partial x} = \frac{l}{v}; \qquad \frac{\partial t}{\partial y} = \frac{m}{v}; \qquad \frac{\partial t}{\partial x} = \frac{n}{v};$$

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2 + \left(\frac{\partial t}{\partial z}\right)^2 = \frac{1}{v^2}.$$

Transforming to polar coordinates and taking the two-dimensional case, we get,

$$\left(\frac{\partial t}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial t}{\partial \theta}\right)^2 = \frac{1}{v^2}.$$

From equation 80 then, along any ray

$$\frac{\partial t}{\partial r} = \pm \frac{1}{r}\sqrt{\frac{r^2}{v^2} - p^2}.$$

Now

$$dt = \frac{\partial t}{\partial \theta}\, d\theta + \frac{\partial t}{\partial r}\, dr.$$

Therefore

$$t = \int p\,d\theta \pm \int \frac{dr}{r}\sqrt{\frac{r^2}{v^2} - p^2}. \tag{81}$$

p is the parameter of the ray. To obtain the equation of a ray we note that a ray is a path of stationary time. Now we may write

$$t_{p+dp} = t_p + \frac{\partial t}{\partial p}\,dp + \frac{\partial^2 t}{\partial p^2}\frac{(dp)^2}{2} + \cdots.$$

For a minimum time path t_{p+dp} must be greater than t_p whether dp is positive or negative; for a maximum time path it must be less; for a saddle point they must be equal (to second orders). Therefore, for a ray,

$$\frac{\partial t}{\partial p} = 0.$$

Applying this to 81, we find

$$\int d\theta = \mp \int \frac{p\,dr}{r\sqrt{\frac{r^2}{v^2} - p^2}}.$$

Take the origin at the center of the earth and $\theta = 0$ at the epicenter. Consider a focus at the surface. Let α be one half the epicentral distance in angular measure. Then

$$\alpha = p\int_{r_d}^{r_0} \frac{dr}{r\sqrt{\frac{r^2}{v^2} - p^2}}, \tag{82}$$

where r_d is the value of r at greatest depth on a ray and r_0 the value at the earth's surface. We assume that the speed is a function of depth only, that is, rays are symmetrical.

Now we observe in seismology a travel time curve which gives p (the quotient of r_0 by the apparent surface speed) as a function of 2α. We desire to know r_d, the greatest depth penetrated by the ray.

At the greatest depth on a ray, $i_d = 90°$,
and

$$\frac{r_d}{v_d} = p = \frac{r_0}{\bar{v}}.$$

Hence the speed at r_d is

$$v_d = \frac{r_d}{r_0}\bar{v},$$

where $\bar{v}$ is obtained directly from the travel time curve, and is the apparent surface speed where the ray emerges.

Equation 82 may be treated by a transformation based on a theorem of Abel and first applied to the seismological problem by Wiechert and Herglotz[1]. This transforms so that the quantities under the integral sign are all known.

If we put

$$\frac{r}{v} = \eta,$$

equation 82 becomes

$$\alpha = p\int_p^{\eta_0} \frac{\frac{d}{d\eta}(\log r)d\eta}{\sqrt{\eta^2 - p^2}},$$

being a function of p. If we write this as

$$\alpha = \int_{\frac{1}{\eta_0^2}}^{\frac{1}{p^2}} \frac{\frac{1}{2}\eta^2\frac{d}{d\eta}(\log r)d\left(\frac{1}{\eta^2}\right)}{\sqrt{\frac{1}{p^2} - \frac{1}{\eta^2}}},$$

it is in practically the same form as that given by Bôcher[2], pages 8–9. The solution is

$$\frac{d}{d\eta}(\log r) = -\frac{2\eta}{\pi}\frac{d}{d\eta}\int_\eta^{\eta_0} \frac{\eta\alpha dp}{p^2\sqrt{p^2 - \eta^2}}. \tag{83}$$

The conditions under which the solution holds[3] are that α as a function of p has only finite discontinuities and that $\alpha = 0$ when $p = \eta_0 = r_0/v_0$. These conditions are satisfied by travel time curves if the focus is at the surface.

However, if the slope of the curve has (if p has) discontinuities, they are difficult to outline and the method is usually used only on smooth (or smoothed!) curves.

The right-hand member of equation 83 may be simplified first, by applying the rule

$$\eta \frac{d}{d\eta} F(\eta) = \frac{d}{d\eta} \eta F(\eta) - F(\eta),$$

where $F(\eta)$ is the integral in our case, and second, by splitting $(d/d\eta)\eta F(\eta)$ in partial fractions, one of which becomes $+F(\eta)$.

Thus 83 becomes

$$\frac{d}{d\eta} \log r = -\frac{2}{\pi} \frac{d}{d\eta} \int^{\eta_0} \frac{\alpha dp}{\sqrt{p^2 - \eta^2}}.$$

Integrating, we get

$$\log \frac{r_0}{r} = -\frac{2}{\pi} \int_{\eta_0}^{\eta} \frac{\alpha dp}{\sqrt{p^2 - \eta^2}}.$$

Before the Abel transformation the integration was along a single ray, the variable being r, with p constant; after the transformation the integration is from ray to ray, the variable is p, and $\eta = r_d/v_d$ (which is the limiting value of p) is the constant.

We integrate along the travel time curve from

$$p = \frac{\partial t}{\partial \theta} = \eta_0 = \frac{r_0}{v_0},$$

that is, the value at epicentral distance zero, to

$$p = \frac{\partial t}{\partial \theta} = \eta = \frac{r_0}{\bar{v}_1} = \frac{r_d}{v_d},$$

the value at some selected epicentral distance Δ. The limits η_0 and η are read from the travel time curve. Corresponding to the Δ chosen we get by solution the r which indicates the maximum depth penetrated by the ray emerging at the selected Δ. We may further write

$$\log \frac{r_0}{r_1} = -\frac{2}{\pi} \int_{\Delta=0}^{\Delta=\Delta_1} \frac{\alpha d \frac{r_0}{\bar{v}}}{\sqrt{\left(\frac{r_0}{\bar{v}}\right)^2 - \left(\frac{r_0}{\bar{v}_1}\right)^2}}$$

$$= -\frac{2}{\pi} \int_{\Delta=0}^{\Delta=\Delta_1} \frac{\alpha d \left(\frac{\bar{v}_1}{\bar{v}}\right)}{\sqrt{\left(\frac{\bar{v}_1}{\bar{v}}\right)^2 - 1}}$$

$$= -\frac{2}{\pi} \int_{\Delta=0}^{\Delta=\Delta_1} \alpha d \cosh^{-1} \left(\frac{\bar{v}_1}{\bar{v}}\right).$$

If we integrate by parts, the first term vanishes. Putting $\alpha = \frac{\Delta}{2r_0}$, we get

$$\log \frac{r_0}{r_1} = \frac{1}{\pi r_0} \int_0^{\Delta_1} \cosh^{-1} \frac{\bar{v}_1}{\bar{v}}\, d\Delta. \tag{84}$$

The integration may be performed graphically. First read the value of $\bar{v}$, the apparent surface speed from the travel time curve for each value of Δ, up to some value $\bar{v}_1$ at Δ_1. Next compute $\cosh^{-1} (\bar{v}_1/\bar{v})$ for frequent values of $\bar{v}$, and plot each value of $\cosh^{-1} (\bar{v}_1/\bar{v})$ against the value of Δ corresponding to the $\bar{v}$. The area under the curve is $\pi r_0 \log (r_0/r_1)$, and r_1 may be evaluated. This gives the distance from the center of the earth to the bottom of the ray emerging at Δ_1, that is, the depth at which the speed $(r_1/r_0)\bar{v}_1$ is attained.

The method is arduous, since such a graphical integration must be performed for each pair of r_1, $\bar{v}_1$ values desired.

Flat earth

For very small epicentral distances (as in local earthquakes or seismic prospecting) we write [after Ewing[4]]

$$r_1 = r_0 - h,$$

where h is the depth of the bottom of the ray from the earth's surface, and note that the case is well approximated by assuming the earth flat ($r_0 = \infty$).

Now

$$\lim_{r_0\to\infty} r_0 \log \frac{r_0}{r_0 - h} = \lim_{r_0\to\infty} h \log \left(\frac{1}{1 - \frac{h}{r_0}}\right)^{\frac{r_0}{h}}$$
$$= \lim_{r_0\to\infty} h \log \left(1 + \frac{h}{r_0}\right)^{\frac{r_0}{h}}$$
$$= \lim_{r_0\to\infty} h \log e$$
$$= h.$$

Hence for a flat earth equation 84 becomes

$$h_1 = \frac{1}{\pi}\int_0^{x_1} \cosh^{-1}\frac{\bar{v}_1}{v}\,dx,$$

where we use x as surface distance.
For a flat earth equation 81 becomes

$$t = \int p\,dx \mp \int \frac{\sqrt{1 - p^2v^2}}{v}\,dz,$$

where now

$$p = \frac{\sin i}{v},$$

which is constant along any ray, and z is the depth measured down from the surface. Setting

$$\frac{\partial t}{\partial p} = 0$$

to get the equation of a ray, we have

$$x = \int_0^z \frac{pv}{\sqrt{1 - p^2v^2}}\,dz. \tag{85}$$

Linear increase of speed with depth. Consider the particular case where the speed v is a linear function of depth:

$$v = v_0 + az. \tag{86}$$

Changing the variable of integration in 85 to v by 86, integrating, and rearranging, we obtain

$$\left(x - \frac{\sqrt{1 - v_0^2p^2}}{ap}\right)^2 + \left(z + \frac{v_0}{a}\right)^2 = \frac{1}{a^2p^2}.$$

Thus a ray is an arc of a circle with center on the line $z = -(v_0/a)$ and radius $1/ap$.

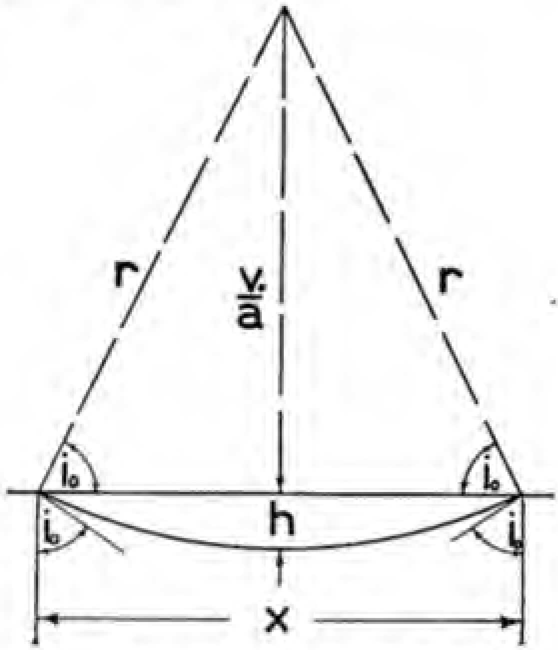

Fig. 42. Circular Ray.

The depth of penetration in terms of the data of the travel time curve is easily obtained by reference to Figure 42.

$$\begin{aligned} h &= r(1 - \sin i_0) \\ &= \frac{x}{2}\frac{1 - \sin i_0}{\cos i_0} \\ &= \frac{x}{2}\sqrt{\frac{\bar{v} - \bar{v}_0}{\bar{v} + \bar{v}_0}}, \end{aligned}$$

since for a flat earth

$$\sin i_0 = \frac{v_0}{v_d}, \qquad v_0 = \bar{v}_0,$$

the speed of waves at the surface and equal to the apparent surface speed very near $x = 0$, and $v_d = \bar{v}$ being the speed at greatest depth on the ray ($\sin i = 1$ here) and equal to the apparent surface speed at x.

The form of travel time curve in such a case of linear increase of speed with depth may be obtained as follows: Referring to Figure 40, the reciprocal of the apparent surface speed at emergence is

$$\left(\frac{dt}{dx}\right)_{x=0} = \frac{\sin i_0}{v_0} = \frac{1}{v_0\sqrt{1 + \frac{x^2a^2}{4v_0^2}}},$$

$$t_x = \frac{1}{v_0}\int_0^x \frac{dx}{\sqrt{1 + \frac{x^2a^2}{4v_0^2}}},$$

where t_x is the time along the surface as plotted on the travel time curve.

$$t_x = \frac{2}{a}\sinh^{-1}\frac{xa}{2v_0}$$

is then the equation of the travel time curve.

Travel time curves with straight line branches. In the study of local earthquakes the travel time curves of the preliminary waves consist of a series of straight lines, similar to the solid portions of the curves in Figure 43. In this figure the x axis represents the surface of the ground. Below it are drawn rays indicating wave paths in the earth. Above are plotted the arrival times corresponding to the various wave paths at the different distances. Branch $ABDC$ marks the arrival of the wave traveling direct from source (at some depth in the earth's upper surface layer) to the observing stations. Since we do not know when or exactly where earthquakes will occur and since the establishment of closely spaced seismographic stations is financially impracticable, it is very rare that the curved portion AB is observed. Only occasionally is the epicentral distance of the seismographic station of the same order as the depth of focus, as it needs to be in order to give a point on AB. The travel time of such waves is given by

$$t_1 = \frac{\sqrt{x^2 + H^2}}{v_1}, \tag{87}$$

where H is the depth of focus of the earthquake and v_1 the speed in the upper layer. As soon as epicentral distance x becomes large compared to H, the curve approximates

$$t_1 = \frac{x}{v_1}.$$

At large distances, say at $x = D'$ in Figure 43, $x = FD'$ quite closely, and the time of occurrence of the shock at the focus is quite accurately

$$t_{D'} - \frac{FD'}{v_1} = t_{D'} - \frac{x_D}{v_1}.$$

Hence straight line branch CB prolonged to the time axis would cut it at the time of occurrence of the earthquake.

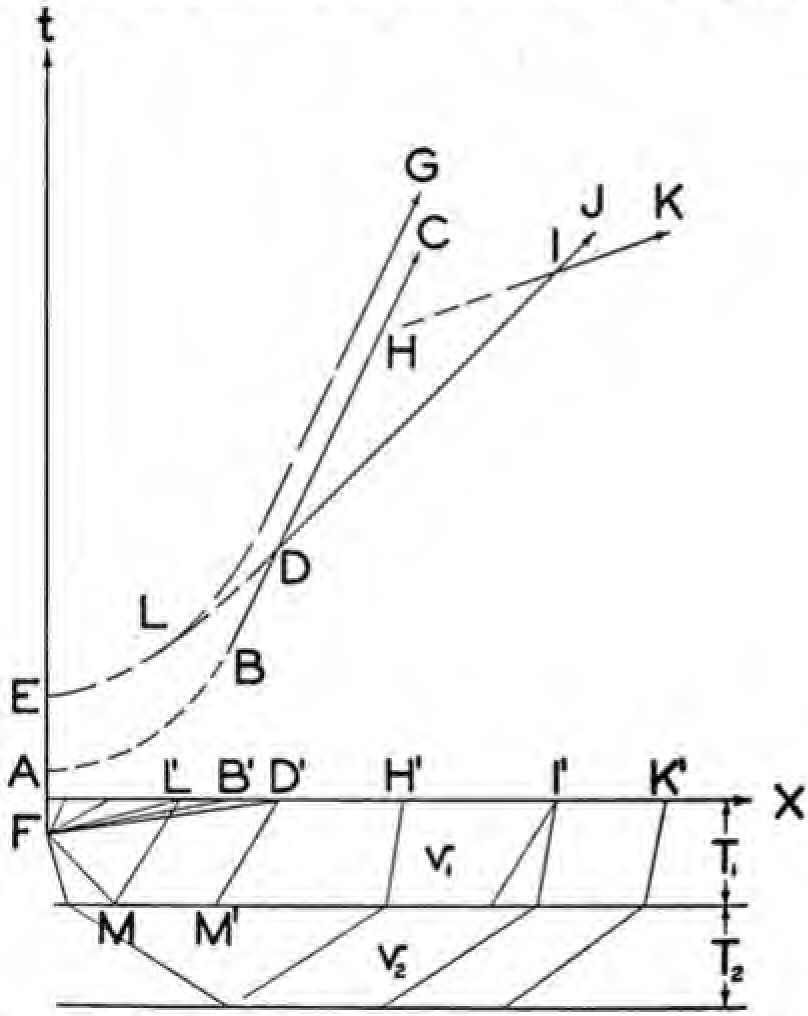

Fig. 43. Wave Paths and Travel Time Curves (diagrammatic).

If there is an observation on branch AB, one may therefore compute H from equation 87, t_1 being the known travel time (arrival time minus time of occurrence at focus), x the epicentral distance, and v_1 the speed in the upper layer (determined from the slope of BC).

Branch *LDJ* of the travel time curve is also straight. It indicates a wave traveling at higher speed than v_1, that is, at speed v_2, and therefore a wave traveling in another medium. Since the observations used in drawing this branch came from stations in different directions from the epicenter, it follows that this second medium is symmetrically placed with respect to the stations and therefore there is indicated an horizontal discontinuity in the earth with a medium of speed v_2 underlying that of v_1. Within our limits of accuracy of observation the branch *DIJ* is a straight line. To the same accuracy the path of the corresponding wave must be a straight, horizontal line in the medium, v_2. Paths of these indirect rays are *FML'*, *FMM'D'*, and so on. They obey Snell's Law but are not strict "optical" paths in that they carry energy, whereas according to geometric optics, energy refracted along the under side of the boundary would not again emerge into the upper medium. Their existence has been proven, I think, by seismic prospectors, who have checked, by boring, the presence of discontinuities computed on the assumption that such paths exist. They have been treated theoretically by solutions of the wave equations by Jeffreys and by Muskat[8].

The indirect wave from the second layer emerges first at *L'*, where it is also the first totally reflected wave. Thus its travel time curve has point *L* in common with branch *ELG*, the travel time curve of the wave reflected at the bottom of the first layer. *ELG* is tangent to *BDC* at infinity, where the reflected wave and the direct wave become one. A second indirect wave, having penetrated a lower medium of speed v_3, emerges first at *H'* and supplies the observations which result in branch *HIK*.

The dotted portions of the travel time curves in Figure 43 are those rarely observed but inferred. The deeper waves are weaker than the more direct ones and rarely noted unless they are first arrivals.

The travel time of the first indirect wave is given by

$$t_2 = \frac{(T_1 - H)\sec i_1'}{v_1} + \frac{\Delta - (T_1 - H)\tan i_1' - T_1 \tan i_1'}{v_2} + \frac{T_1 \sec i_1'}{v_1}$$

$$= \frac{\Delta}{v_2} + (2T_1 - H)\frac{\cos i_1'}{v_1},$$

where the subscript on the symbol i indicates that the angle is measured in the first layer (of thickness T_1) and the prime indicates that the wave travels horizontally just below the bottom of the first layer $\sin i_1' = \frac{v_1}{v_2}$. The slope of branch DIJ gives the speed v_2.

The travel time of the second indirect wave is

$$t_3 = \frac{\Delta}{v_3} + (2T_1 - H)\frac{\cos i_1''}{v_1} + 2T_2\frac{\cos i_2''}{v_2},$$

$$\sin i_1'' = \frac{v_1}{v_3},$$

$$\sin i_2'' = \frac{v_2}{v_3},$$

and so on for as many layers as there are branches of the travel time curve less one.

Earth structure

Surface layering. It appears from studies of local earthquakes that the earth's crust in localities studied consists of at least two layers, possibly three, of total thickness of 30 to 40 kilometers. The upper layer in most localities consists of a medium in which P waves (called $\bar{P}$ in this layer) show a speed of about 5.6 km/sec. This is usually referred to as the *granitic* layer and has a thickness of 10 to 15 kilometers. Below the layering is the *mantle,* at the upper surface of which P waves (sometimes called P_n waves) have a speed of about 8.0 km/sec (see Figure 44).

In some regions the speed in the upper layer seems to be higher than 5.6 km/sec, for example in New England. The exact thickness and speed in lower crustal layers is

difficult to obtain. The waves refracted along the top of one of these layers rarely are recorded as first arrivals. The reading of the beginning of the wave groups is difficult,

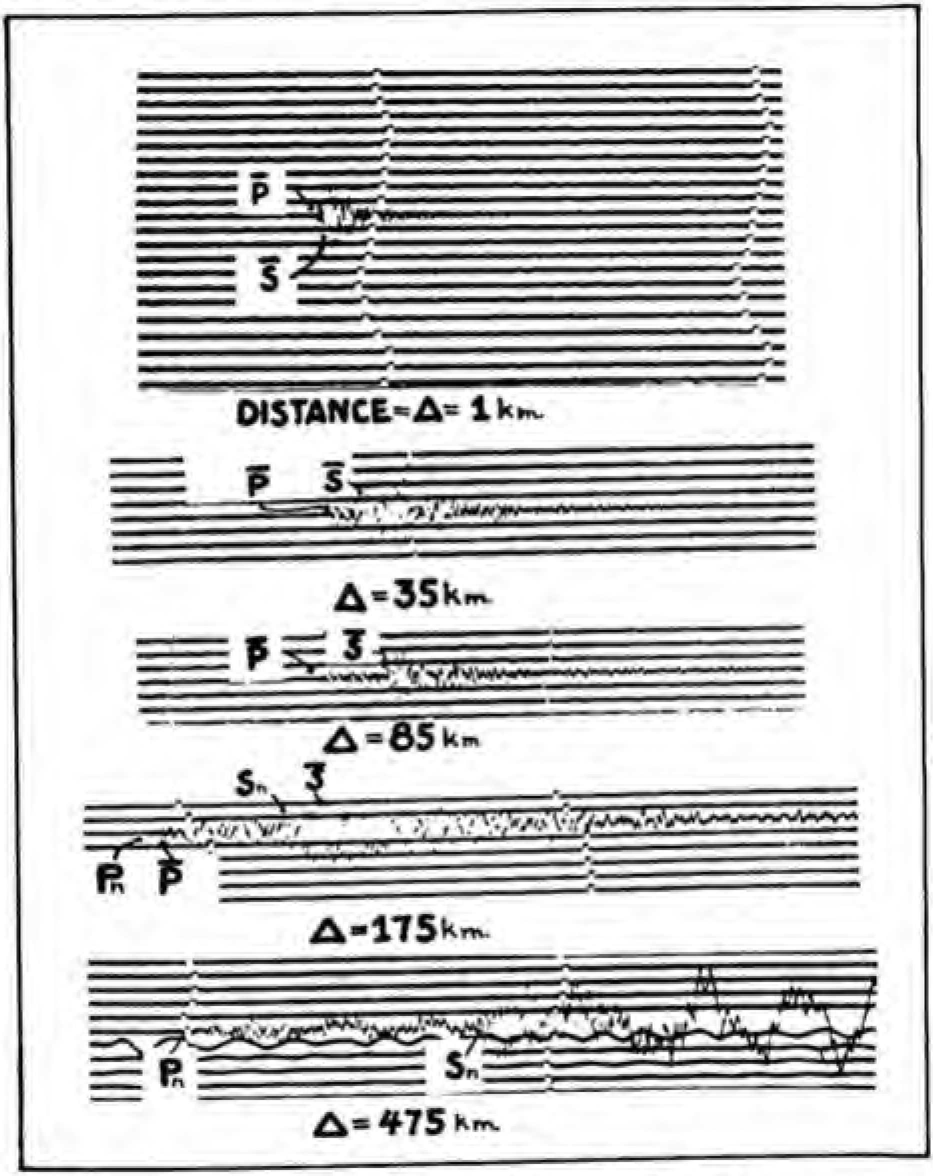

Fig. 44. Seismograms of Near Earthquakes.

the points on the travel time graph are scattered, and personal judgment enters into the construction of the travel time curves. $\bar{P}$ and P_n (P in the mantle) are both first arrivals in appreciable ranges of distance. The con-

struction of their travel time curves is relatively free of uncertainty.

Waves reflected at the boundaries between the crustal layers have not been definitely observed for earthquakes. It is necessary to trace the travel time curve for a given type of wave before it can be said to be identified. After such a travel time curve has once been traced, future individual observations may be identified with it. The difficulty with reflected waves is that many stations very near the epicenter would be needed to trace the reflections clearly. These are not available. If we knew when and where earthquakes would occur, we could set up temporary stations for such observation, but that would require an equipment of portable instruments probably not available at any seismographic station. Again, the lower intensity of reflected waves may very well allow them to be masked by surface waves, and we might have to resort to filtering as in seismic prospecting.

The mantle. The portion of the earth below the surface layers and above the core (which is first encountered at a depth of about 2,900 kilometers) is called the *mantle.* The travel time curve of P waves which have the deepest portion of their paths in the mantle has usually been interpreted as smooth. There are a number of investigators, however, who feel that the curve has been smoothed rather than is smooth. Several studies of individual earthquakes have suggested that the travel time curve for P in the mantle might well be drawn as a series of almost straight branches. The first of these branches ends at an epicentral distance of about 20°, and here the slope takes a sharp change. Whether or not this change is discontinuous has not yet been established. If it is shown to be so, many laborious integrations of smoothed curves made in the past will be outmoded. Referring to Figure 45, (a) shows the travel time for a continuous moderate increase of speed with depth; (b) shows the curve for the case in which at some depth the rate of change of speed with depth becomes very

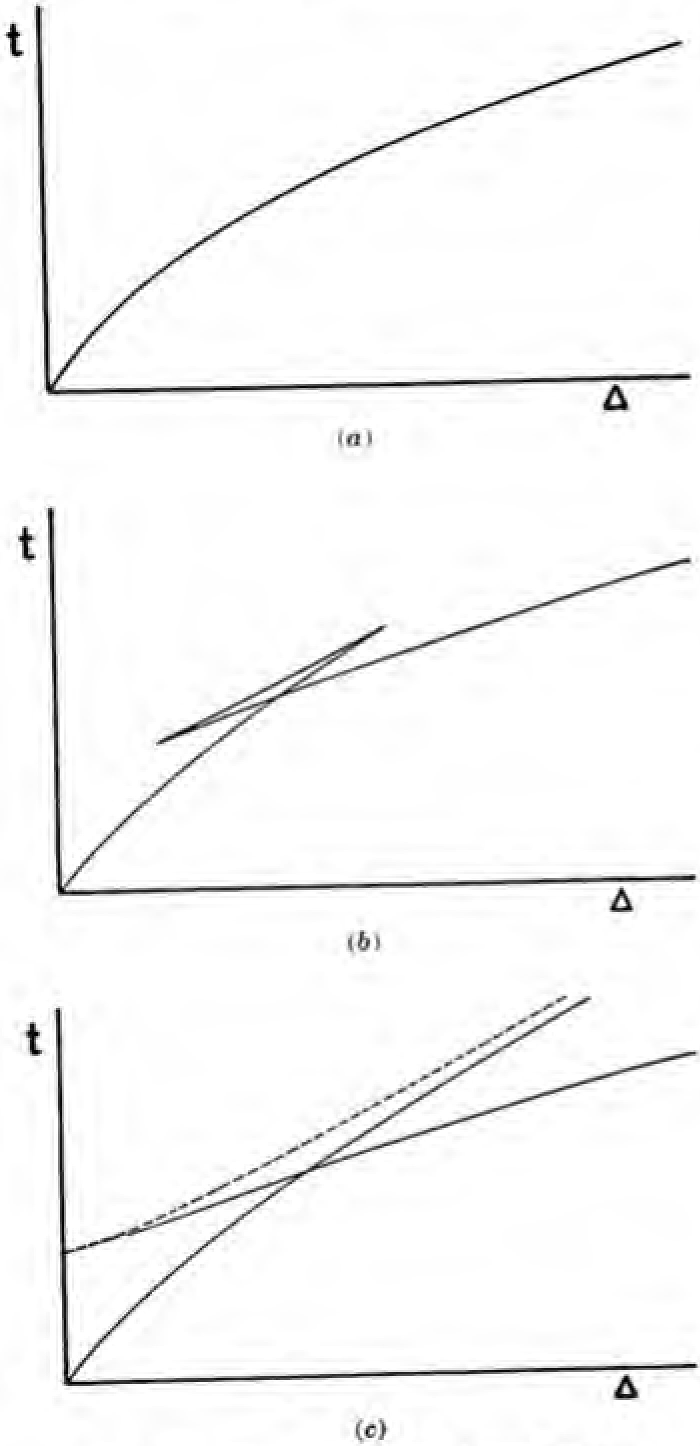

Fig. 45. Types of Travel Time Curves.

rapid for a while, although still continuous; (c) represents the case of a discontinuous change in speed at a certain depth, the dotted curve corresponding to a reflected wave. Since the P group is quite irregular, it is difficult to identify certainly a second arrival shortly after the first and to determine whether or not the complications of (b) and (c) exist. It is too easy to see second waves on all records.

Basing his conclusions on his own P curve and a gradual increase of speed with depth below the surface layers,

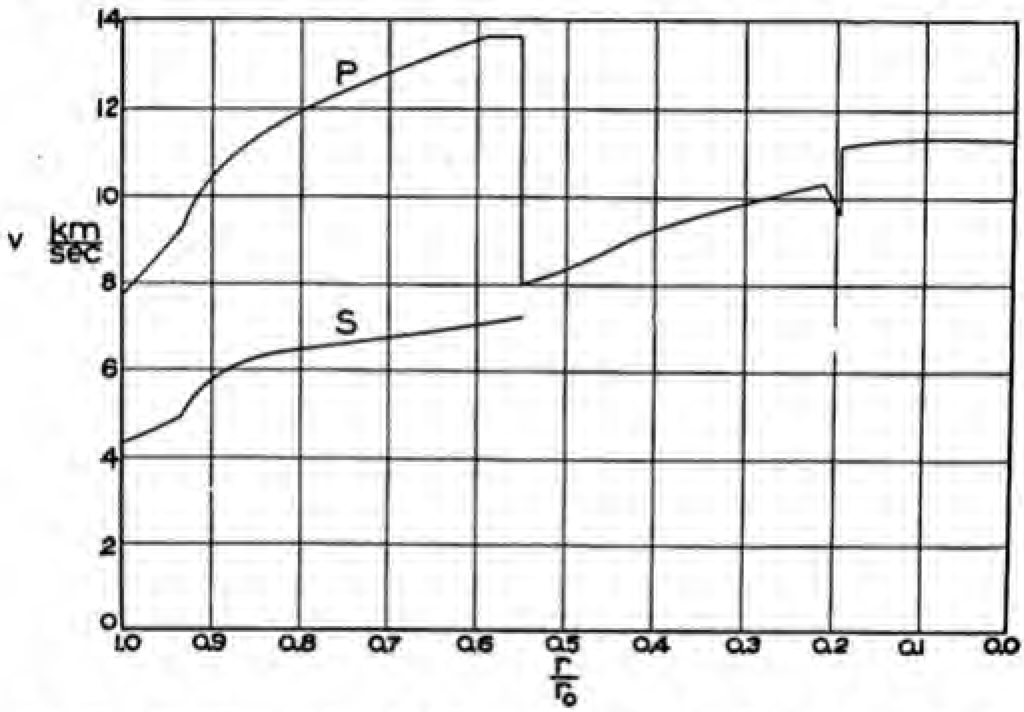

Fig. 46. Variation of Speed of Seismic Waves with Depth in the Earth

Jeffreys[5] finds the wave emerging at about 20° to have penetrated to a depth of about 480 kilometers. It would be, then, at about this depth that some change occurs in the earth's composition or in the rate of change with depth of its composition. Integrations of smoothed travel time curves have led to the conclusion of a change at a depth near 1,000 kilometers, and perhaps some others below it[9].

Figure 46 gives the speed of P and S waves as a function of depth in the earth, after Jeffreys (r is distance from center of earth, and r_0 the radius of the earth).

The core. At a depth in the earth of about 2,900 kilometers there is a change in its constitution. This discontinuity reflects waves of length at least as short as 100 kilometers, probably considerably shorter. Therefore, the change takes place in less than 50 kilometers, probably considerably less. *P* waves are refracted sharply at this level, at which their speed drops. This drop in speed results in a shadow zone for *P* as indicated in Figure 47.

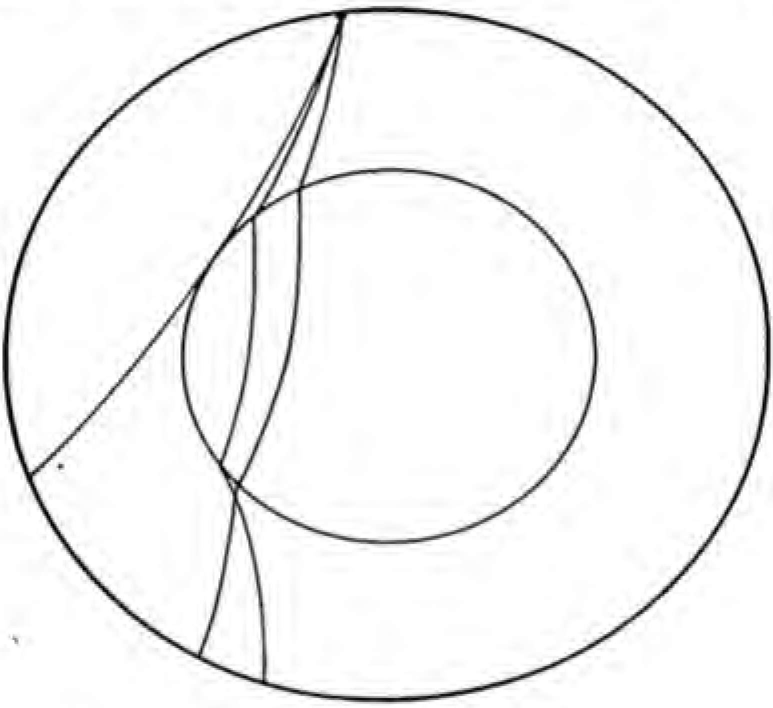

Fig. 47. The Shadow Zone.

It has never been demonstrated satisfactorily that *S* waves traverse the core. When *P* arrives through the core, it is the first wave and disturbs quiet, but when *S* is due through the core, the record has been disturbed by other earlier waves, and the positive identification of a weak wave is doubly difficult. At this depth in the earth the pressure far exceeds any attainable at the earth's surface, and the temperature is unknown. Many would prefer not to use the words "solid" and "fluid" to describe matter under such conditions, which are not attainable in the laboratory.

But if we wish to use these words, we may define a fluid as matter in a state which will not resist shear of magnitude and frequency similar to that exerted by S waves, and a solid as matter in a state which will resist such shear. Then, since it cannot be shown that shear waves traverse the core, whereas it can be shown that they traverse the mantle down to the core by paths similar to those of compression waves, and also that compression waves traverse the core, it follows that the core is fluid as far as our knowledge goes. One must remember that any test "in the laboratory" which attempts to distinguish between the solid and liquid states must adopt some such definition as that assumed here for the core. It is idle to say that perhaps S waves do travel through the core, but that we cannot identify them. Recent developments indicate that the core is also layered. According to Gutenberg and Richter[6], these are second-order discontinuities where the acceleration of P waves, not the speed, changes abruptly with depth.

P and *S*

An earth of such complicated structure gives rise to many groups of waves. There are $\bar{P}$ and $\bar{S}$ traveling in the upper surface layer, which are recorded only at near stations, since the curvature of the earth shields distant stations from them. Of course the focus must be in the upper layer for these waves to exist. There are P^* and S^*, which travel the horizontal portion of their paths in the "intermediate" layer. The multitude of arrivals of apparently new groups of waves in near-by earthquakes have led some investigators to postulate still more surface layers. The normal P and S waves at stations not very near the epicenter are the P and S usually referred to in discussions. They travel down through the layers from the focus, traverse the mantle, and again emerge through the surface layers. Their paths are least time paths for longitudinal and transverse waves respectively. It is at epicentral distance

of about 103° that the paths of P and S (almost the same) graze the core. The core casts a shadow on the region just beyond $\Delta = 103°$. Beyond 103° the P travel time curve is a straight line represented on the records by a weak wave diffracted along the boundary.

Around 110° there begins, some four minutes after the diffracted P, a longitudinal wave which has passed into the complex core. This is called P'. Its travel time curve is cusped, according to Gutenberg and Richter, owing to the variable velocity distribution in the core. As usual in such cases the observed points on the travel time curve are somewhat scattered and the existence of two branches near a cusp must be largely inferred on theoretical grounds. P' is observed in its several branches as far as the anticenter (180° from the epicenter). Near 142° it is very strong.

Reflected waves

Two discontinuities of the earth have been definitely recognized as reflectors of seismic waves, the surface of the earth and the core boundary. At these boundaries P waves are reflected both as P and S, and S waves are reflected both as S and P. The P waves are reflected on the inside of the core boundary as well as on the outside. Thus very many groups of waves on the seismogram are explained as reflections. An early nomenclature proposed to describe these waves was constructed as follows: Each portion of the path of a wave was given as the capital letter, P or S, depending on the nature of the vibration on that portion of the path. The letters were put in consecutive order corresponding to the segments of the path proceeding from the focus to the station. If the transformation from one branch to the other was by refraction, a bar was placed over the two letters; if by reflection, no bar was used. If the transformation was at the core boundary, a subscript c was placed between the two letters designating the nature of vibration before and after transformation. If the transformation (reflection) was at the

earth's surface, no subscript was used. Thus PP designates a wave leaving the focus as P and reflected at the earth's surface as P at a point about midway between epicenter and station. PS indicates a surface reflection which was P on the first branch of its path and S on the second. PPP is P reflected twice at the earth's surface. $\overline{S_cP_c}\,\overline{P_cS}$ is a wave leaving the focus as S, refracted at the core boundary as P, reflected then at the interior side of the core boundary

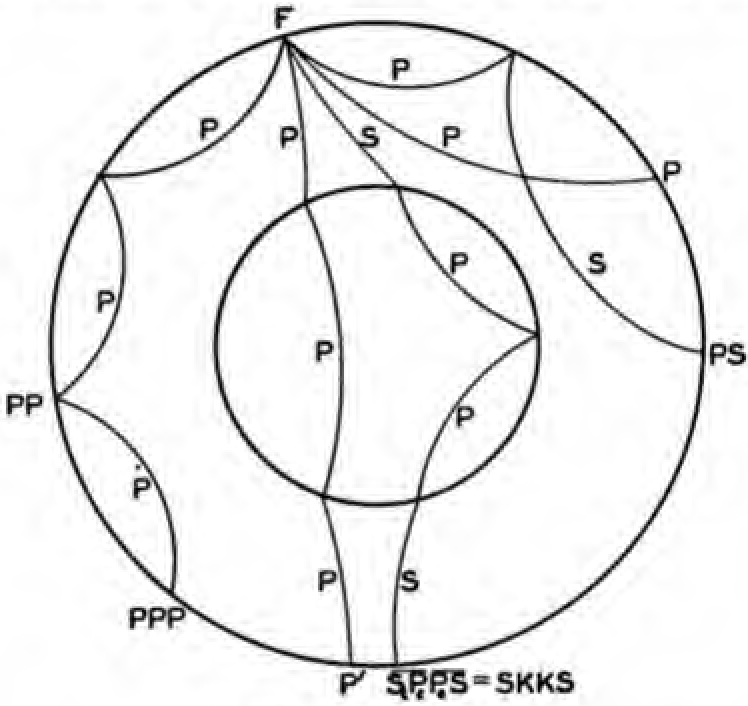

Fig. 48. Paths of Waves.

as P, and finally refracted out of the core as S. Some of the waves now identified, in addition to those mentioned, are SS, SSS, P_cP, S_cS, S_cP, $\overline{S_cP_cP}$, $\overline{P_cP_cP} = P'$, $\overline{P_cP_cP}\,\overline{P_cP_cP} = P'P'$. Another nomenclature drops the bars and subscripts and uses the letter K for a branch of path (as P) in the core. So $\overline{S_cP_cS} = SKS$. Many of the waves through the core are recognized after having been reflected once at the earth's surface. Figure 48 indicates some of these paths. Figure 49 shows travel time curves for the various waves, based on recent tables of Jeffreys and Bullen[7].

For deep-focus earthquakes there become important another group of reflected waves. These are reflected very near the epicenter, and for them the first branch of the curve

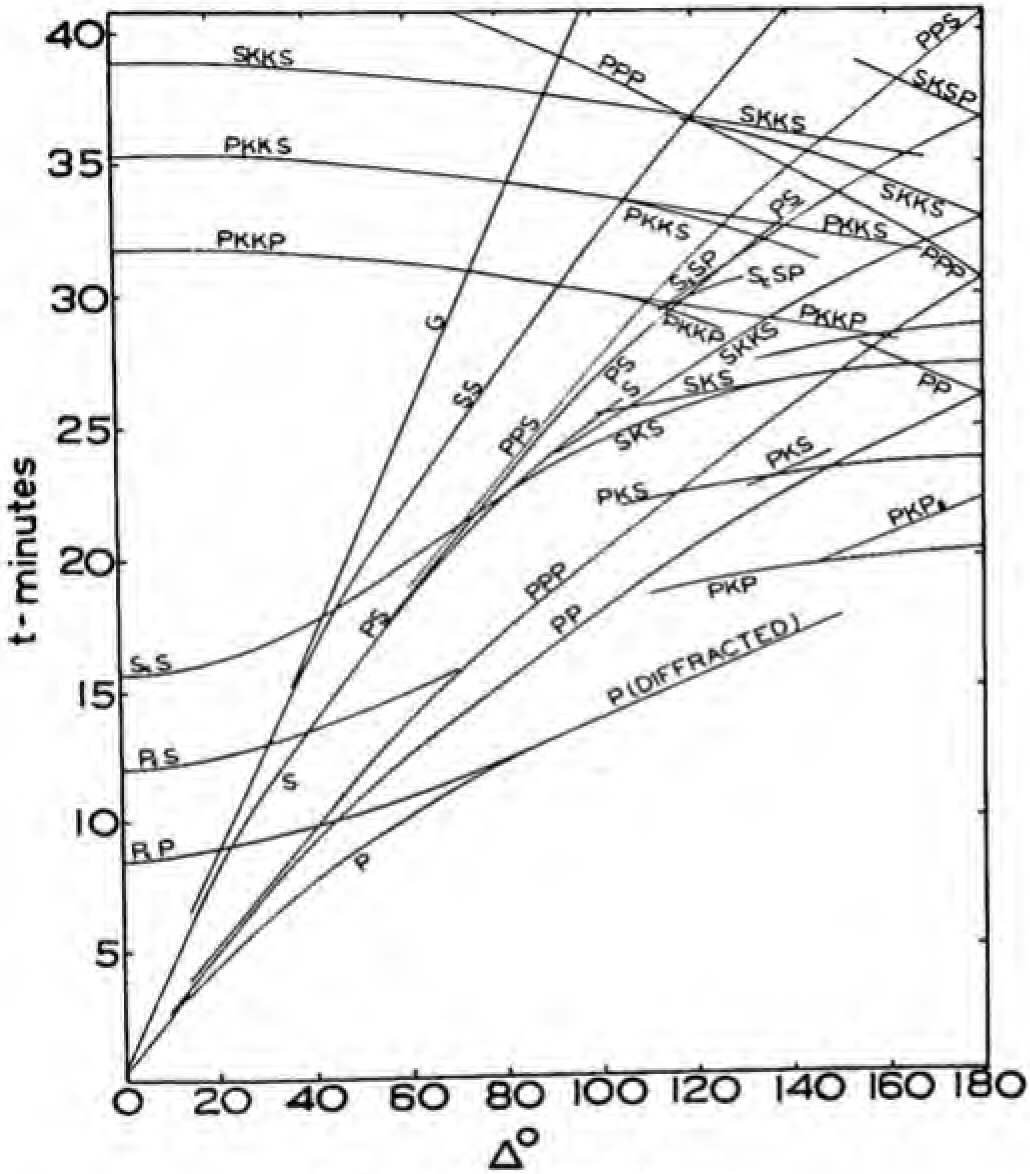

Fig. 49. Travel Time Curves.

is designated by a small *p* or *s*, depending on the nature of the vibration on that branch. Thus *pP* represents the *P* wave going nearly vertical from the focus to the surface

and thence reflected as P to a distant station. This wave may also strike the core and undergo other reflections and refractions. Even in shallow earthquakes pP no doubt exists but is so confused with the direct P as to be very difficult to identify. The interval between P and pP is very useful in determining depth of focus.

Paths of reflected waves in a homogeneous sphere

The possibilities of reflection within a layered earth are manifold. Reflection at curved boundaries offers complications not observed in plane reflections.

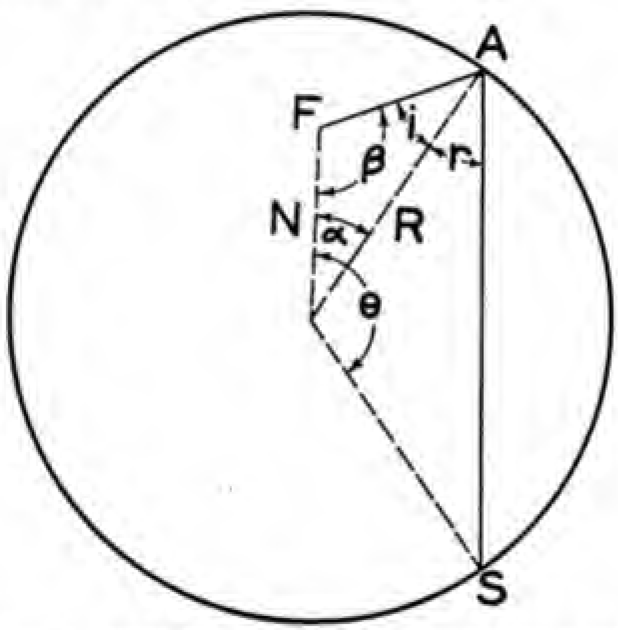

Fig. 50. Rays Reflected in a Homogeneous Earth.

For a simplified example let us consider a source in a homogeneous sphere with an observing station on the surface; that is, assume the earth homogeneous. Study of this problem will lead us to a better understanding of some of the complexities of the seismogram. In Figure 50 let F be the focus of the shock, N its distance from the earth's center, and R the earth's radius. FAS is the path of a reflected wave. Now, according to Huyghens' Principle, all points on a wave front act as new sources, and *a priori* we might expect all points on the reflecting surface of the

sphere to send energy to the station. However, Fermat's Principle states that of all such possible paths only those are optical paths which satisfy the equation $\partial t/\partial\alpha = 0$, where t represents the time and α a suitable parameter, in our case the angle so marked in Figure 50. The statement that certain paths are optical paths means that the time elapsing between the shock and the arrival of the beginning of a definite group of waves at the station may be obtained by dividing the distance along the optical path by the speed of waves in the medium, as computed from its elastic constants. The partial derivative indicates that we shall hold the epicentral distance a constant. The vanishing of $\partial t/\partial\alpha$ indicates a stationary value of t as function of α. This stationary value may be a maximum (for PP) or a minimum (for pP) or merely a saddle value. It is the case where two adjacent possible paths (two with α varying only infinitesimally) have identical travel times and are therefore able to reinforce each other.

We now express the time as a function of θ (angular epicentral distance of observing station), which we shall hold constant, and α, which we shall vary.

$$t = \frac{FA}{v} + \frac{AS}{v},$$

where we consider P waves reflected as P waves or S waves reflected as S waves, and not the transformed types.

$$vt = \sqrt{N^2 + R^2 - 2NR\cos\alpha} + 2R\sin\frac{\theta - \alpha}{2}.$$

The sign of this radical is always positive, for it is distance FA.

$$v\frac{\partial t}{\partial\alpha} = \frac{NR\sin\alpha}{\sqrt{N^2 + R^2 - 2NR\cos\alpha}} - R\cos\frac{\theta - \alpha}{2}.$$

Setting $\dfrac{\partial t}{\partial\alpha} = 0$, we have

$$\frac{N\sin\alpha}{\sqrt{N^2 + R^2 - 2NR\cos\alpha}} = \cos\frac{\theta - \alpha}{2},$$

or

$$\sin i = \sin r.$$

That is, the optical paths will be those which make the angles of reflection equal to the angles of incidence. How many such paths are there to a given station? From the figure,

$$3i = 360° - \theta - \beta,$$

where i is considered negative for $360° > \beta > 180°$. Putting $360° - \theta = \kappa$, we write

$$\sin i = \sin \frac{\kappa - \beta}{3} = \frac{N}{R} \sin \beta,$$

and in so doing double the number of roots for β as a function of κ by introducing false roots which require values of i lying in second and third quadrants. These are not allowed physically. That is, for every value of i, i_1, which lies between $-90°$ and $+90°$ and satisfies our problem, we introduce another, $180° - i_1$. Then when we set a value of $\beta = \beta_1$ and get $(N/R) \sin \beta_1$, we have to compute $(\kappa - \beta_1)/3$, which may be either $(\kappa_1 - \beta_1)/3$ or $180° - (\kappa_1 - \beta_1)/3$; thus for a given β we get two values of κ, one of which is false; therefore we get a false θ. When we substitute the false θ in our final equation, we shall get β_1, which will be false in this case.

$$\begin{aligned}\frac{N}{R} \sin \beta &= \sin \frac{\kappa - \beta}{3} \\ &= \frac{1}{3} \sin (\kappa - \beta) + \frac{4}{3} \sin^3 \frac{\kappa - \beta}{3} \\ &= \frac{1}{3} \sin (\kappa - \beta) + \frac{4}{3} \frac{N^3}{R^3} \sin^3 \beta\end{aligned}$$

$$-4 \frac{N^3}{R^3} \sin^3 \beta + \left(3 \frac{N}{R} + \cos \kappa\right) \sin \beta = \sin \kappa \cos \beta.$$

We must square this to get an equation in $\sin \beta$ only and thus again double the number of roots, since this operation introduces the last equation with $- \cos \beta$ substituted for

$\cos \beta$. Squaring leads to

$$16\frac{N^6}{R^6}\sin^6\beta - 8\frac{N^3}{R^3}\left(3\frac{N}{R} + \cos\kappa\right)\sin^4\beta + \left(9\frac{N^2}{R^2} + 6\frac{N}{R}\cos\kappa + 1\right)\sin^2\beta - \sin^2\kappa = 0,$$

and we get four times as many roots of β as are pertinent.

For all reasonable values of N/R (in earthquakes it is rarely less than 0.9) the quantities in parentheses are positive, and by Descartes Rule the equation considered as third degree in $\sin^2\beta$ has no negative roots. Therefore we have three positive values of $\sin^2\beta$ and, therefore, twelve of β, of which 9 are false.

Graphical solution for values of θ not too small indicate that the three roots correspond to pP, PP, and PP by the arc greater than 180°. These correspond to one value of β between 90° ca. and 180°, one between 45° ca. and 90° ca., and one between 270° ca. and 360° ca. The critical value of β between pP and PP is actually somewhat less than 90°, as is shown below. pP cannot be observed reflected on the arc greater than 180°. This would required $2i < \alpha$, $3i < 180° - \beta$.

Now

$$\sin i = \frac{N}{R}\sin(180° - \beta),$$

and

$$\frac{di}{d(180° - \beta)} = \frac{N}{R}\frac{\cos(180° - \beta)}{\sqrt{1 - \frac{N^2}{R^2}\sin^2(180° - \beta)}}.$$

At $180° - \beta = 0$, reflection at the epicenter,

$$\frac{R}{N}\,di = d(180° - \beta) < 3\,di\left(\text{if } N > \frac{R}{3}\right)$$

and

$$i = 180° - \beta = 0.$$

$di/d(180° - \beta)$ has its maximum at this point and is positive; i and $180° - \beta$ increase together, and $180° - \beta$ increases more rapidly than i as far as $\beta = 90°$. However, $di/d(180° - \beta)$ does not get as low as $\frac{1}{3}$ [that is, 3 di does not get as small as $d(180° - \beta)$] until

$$\cos (180° - \beta) = +\sqrt{\frac{R^2 - N^2}{8N^2}},$$

and, as shown below, this value corresponds to the outer bound of pP as the term is used here; reflections for larger values of $180° - \beta$ are maximum rather than minimum time paths.

To ascertain which of the paths obeying Snell's Law (pP and PP) are maximum, minimum, or saddle values of the time, we examine the second derivative:

$$v\frac{\partial^2 t}{\partial \alpha^2} = \frac{N(R) \cos \alpha}{\sqrt{N^2 + R^2 - 2NR \cos \alpha}} - \frac{(NR \sin \alpha)^2}{(\sqrt{N^2 + R^2 - 2NR \cos \alpha})^3} - \frac{R}{2} \sin \frac{\theta - \alpha}{2}.$$

For α very small (θ near 180°)

$$v\frac{\partial^2 t}{\partial \alpha^2} = \frac{3NR - R^2}{2(R - N)},$$

which is positive for $N > \frac{R}{3}$.

Therefore, if the depth of focus is less than two thirds of the radius of the sphere, there will be waves of small α reflected near the epicenter which will arrive near the anticenter by minimum time paths. These paths will not be minimum but maximum if the focus is deeper than two thirds the radius of the sphere, and for the particular case of $N = R/3$ the path to the anticenter will be due to a saddle value in the time.

Again, if α approaches 90° (θ will lie between 90° and 270° and will lie near 180° for small depth of focus),

$$v\frac{\partial^2 t}{\partial \alpha^2} = -\frac{N^2R^2}{(\sqrt{N^2 + R^2})^3} - \frac{R}{2} \sin \frac{\theta - 90°}{2}.$$

This value is negative. There will arrive near the anticenter, if the depth of focus is not too great, a second reflected wave, one reflected near the midpoint between epicenter and anticenter, and it will travel a maximum time path.

The minimum time waves are called *pP* (or *sS*), and the maximum time paths *PP* (or *SS*). At a value of α, call it α_c, between 0° and 90°, $\partial^2 t/\partial\alpha^2$ must pass through zero. At the epicentral distance, θ_c, corresponding to this value of α, *pP* and *PP* will arrive together, and the path will be one of a saddle value of the time, neither maximum nor minimum. This occurs when

$$\cos \alpha_c = \frac{R^2 + 3N^2}{4RN},$$

$$\cos i_c = \frac{3}{2\sqrt{2}} \frac{\sqrt{R^2 - N^2}}{R} = \sin \frac{\theta_c - \alpha_c}{2},$$

$$\cos \beta_c = -\sqrt{\frac{R^2 - N^2}{8N^2}},$$

$$\overline{FA}_c = \frac{\sqrt{R^2 - N^2}}{\sqrt{2}},$$

$$\overline{AS}_c = \frac{3\sqrt{R^2 - N^2}}{\sqrt{2}},$$

and

$$\overline{FA}_c + \overline{AS}_c = \sqrt{8(R^2 - N^2)} = vT_c.$$

We shall call these values "critical." For depths of focus less than two thirds the radius, if α is less than the critical value, α_c, the wave reflected is one of minimum time, *pP*. If α exceeds α_c, the wave reflected is one of maximum time, *PP*.

Now

$$\frac{d(\cos \alpha_c)}{dN} = \frac{3N^2 - R^2}{4RN^2}$$

is positive if $N > \dfrac{R}{\sqrt{3}}$.

So as depth of focus increases from zero to $N = R/\sqrt{3}$, α_c increases from zero to 30°; then as the depth increases further, α_c decreases until, at $N = R/3$, α_c is zero. For depths still greater there is no inner zone near the epicenter at which the reflected time is a minimum (no pP wave).

Now

$$\sin i = \frac{N}{R} \sin \beta,$$

$$\frac{di}{d\beta} = \frac{N}{R} \frac{\cos \beta}{\sqrt{1 - \frac{N^2}{R^2} \sin^2 \beta}},$$

and

$$\frac{d^2 i}{d^2\beta} = \frac{N}{R} \left\{ \frac{-\sin \beta}{\sqrt{1 - \frac{N^2}{R^2} \sin^2 \beta}} + \frac{N^2}{R^2} \frac{\cos^2 \beta \sin \beta}{\left(1 - \frac{N^2}{R^2} \sin^2 \beta\right)^{\frac{3}{2}}} \right\}.$$

The radicals are positive because $i < 90°$; i has a stationary value when $di/d\beta = 0$ or $\beta = 90°$. Then

$$\frac{d^2 i}{d\beta^2} = -\frac{N}{R \cos i},$$

so we get a maximum for i. This is the ray which leaves the focus horizontally.

We may write

$$\begin{aligned} vt &= R \frac{\sin \alpha}{\sin \beta} + 2R \cos i \\ &= N \cos \beta + 3R \sqrt{1 - \frac{N^2}{R^2} \sin^2 \beta}. \end{aligned}$$

$$\cos \beta = -\frac{vt}{8N} + \frac{3}{8N} \sqrt{v^2 t^2 - 8(R^2 - N^2)}.$$

Thus $\cos \beta$ is real if

$$v^2 t^2 \geq 8(R^2 - N^2).$$

When

$$\cos \beta = -\sqrt{\frac{R^2 - N^2}{8N^2}} = \cos \beta_c,$$

$$t = \frac{\sqrt{8(R^2 - N^2)}}{v}.$$

This equation gives the least value the travel time may have and occurs at the critical distance mentioned above. For larger values of time there are, for every value of t, two values of the β, one less and one greater than β_c. Thus there are two branches to the travel time curve, one for pP and a later one for PP, which meet in a cusp at

$$t = \frac{\sqrt{8(R^2 - N^2)}}{v}.$$

Now the apparent surface speed of the emerging reflected wave is

$$\bar{v} = R\left(\frac{d\theta}{dt}\right)_{r=R} = \frac{v}{\sin i},$$

$$\left(\frac{d\theta}{dt}\right)_{r=R} = \frac{v}{R \sin i}.$$

So the slope of the travel time curve, $dt/d\theta$, is always positive. Moreover, it is maximum when i is maximum (at the distance of emergence of the ray which leaves the focus horizontally, that is, at a point on the PP or upper branch at an epicentral distance a little greater than that corresponding to the cusp).

Tables of travel times

There are a number of good travel time tables for the various waves. An abbreviated table follows, based on the 1940 tables of Jeffreys and Bullen[7]. The times for G are based on an average speed of 4.35 km/sec. O represents the time of occurrence.

Travel Time Tables
Surface Focus

Δ	*P-O*	*S-P*	*PP-P*	*SS-S*	*G-S*	Δ	*P-O*	*S-P*	*PP-P*	*SS-S*	*G-S*
	m s	*m s*	*m s*	*m s*	*m*		*m s*	*m s*	*m s*	*m s*	*m*
1°	21	15	7	11		41°	7-46	6-13	1-38	2-57	3.5
2°	35	26	7	10		42°	7-55	6-20	1-40	3-01	3.7
3°	50	37	7	11		43°	8-03	6-26	1-42	3-05	3.8
4°	1-04	48	7	11		44°	8-11	6-33	1-44	3-09	4.0
5°	1-18	59	7	10		45°	8-19	6-39	1-46	3-13	4.2
6°	1-32	1-10	7	11		46°	8-27	6-45	1-48	3-17	4.4
7°	1-46	1-21	8	11		47°	8-35	6-52	1-50	3-19	4.6
8°	2-00	1-32	8	12		48°	8-43	6-58	1-51	3-23	4.8
9°	2-14	1-43	8	12		49°	8-50	7-04	1-54	3-26	5.0
10°	2-28	1-54	8	13		50°	8-58	7-11	1-56	3-29	5.2
11°	2-42	2-05	8	13		51°	9-06	7-17	1-57	3-33	5.4
12°	2-55	2-16	10	14		52°	9-13	7-23	1-59	3-35	5.6
13°	3-09	2-27	10	15		53°	9-21	7-29	2-01	3-38	5.8
14°	3-22	2-37	11	16		54°	9-28	7-35	2-03	3-41	6.0
15°	3-35	2-48	12	17		55°	9-35	7-41	2-05	3-43	6.2
16°	3-48	2-58	13	19		56°	9-43	7-48	2-06	3-46	6.4
17°	4-01	3-09	14	20	.1	57°	9-50	7-54	2-08	3-50	6.6
18°	4-13	3-19	15	23	.1	58°	9-57	8-00	2-10	3-52	6.8
19°	4-26	3-29	16	25	.2	59°	10-04	8-06	2-12	3-55	7.0
20°	4-37	3-40	19	28	.2	60°	10-11	8-12	2-14	3-57	7.2
21°	4-47	3-50	23	32	.3	61°	10-18	8-18	2-16	4-01	7.4
22°	4-58	3-59	25	38	.4	62°	10-24	8-24	2-19	4-04	7.6
23°	5-07	4-07	30	44	.6	63°	10-31	8-30	2-20	4-07	7.8
24°	5-17	4-15	34	50	.7	64°	10-38	8-36	2-22	4-10	8.1
25°	5-27	4-22	37	58	.8	65°	10-44	8-42	2-25	4-13	8.3
26°	5-36	4-29	41	1-05	1.0	66°	10-50	8-47	2-28	4-16	8.5
27°	5-45	4-37	46	1-13	1.1	67°	10-57	8-53	2-29	4-20	8.7
28°	5-55	4-44	49	1-21	1.3	68°	11-03	8-59	2-32	4-23	8.9
29°	6-04	4-51	53	1-28	1.4	69°	11-09	9-05	2-35	4-27	9.2
30°	6-13	4-58	57	1-36	1.6	70°	11-15	9-10	2-37	4-30	9.4
31°	6-21	5-05	1-02	1-43	1.8	71°	11-22	9-16	2-39	4-35	9.6
32°	6-30	5-12	1-06	1-51	2.0	72°	11-28	9-21	2-41	4-38	9.9
33°	6-39	5-18	1-10	1-59	2.1	73°	11-33	9-27	2-45	4-43	10.1
34°	6-48	5-25	1-13	2-06	2.3	74°	11-39	9-32	2-47	4-47	10.3
35°	6-56	5-32	1-18	2-14	2.4	75°	11-45	9-38	2-50	4-50	10.5
36°	7-05	5-39	1-21	2-21	2.6	76°	11-51	9-43	2-52	4-55	10.8
37°	7-13	5-46	1-26	2-28	2.8	77°	11-56	9-48	2-55	4-59	11.1
38°	7-21	5-53	1-30	2-36	3.0	78°	12-02	9-54	2-58	5-04	11.3
39°	7-30	6-00	1-33	2-43	3.1	79°	12-07	9-59	3-01	5-08	11.5
40°	7-38	6-06	1-36	2-49	3.3	80°	12-13	10-04	3-03	5-12	11.8

SURFACE FOCUS.—(*Continued*)

Δ	*P-O*	*S-P*	*PP-P*	*SS-S*	*G-S*	Δ	*P-O*	*S-P*	*PP-P*	*SS-S*	*G-S*
	m s	*m s*	*m s*	*m s*	*m*		*m s*	*m s*	*m s*	*m s*	*m*
81°	12-18	10-09	3-07	5-17	12.0	93°	13-17	11-05	3-45	6-18	15.2
82°	12-23	10-14	3-10	5-22	12.3	94°	13-21	11-09	3-49	6-23	15.5
83°	12-28	10-19	3-13	5-27	12.6	95°	13-26	11-13	3-51	6-29	15.8
84°	12-34	10-24	3-15	5-31	12.8	96°	13-30	11-16	3-55	6-34	16.1
85°	12-39	10-29	3-18	5-36	13.1	97°	13-35	11-20	3-58	6-40	16.4
86°	12-44	10-34	3-21	5-41	13.3	98°	13-39	11-24	4-02	6-45	16.6
87°	12-48	10-38	3-26	5-45	13.6	99°	13-44	11-28	4-04	6-51	16.9
88°	12-53	10-43	3-29	5-51	13.9	100°	13-48	11-32	4-08	6-57	17.2
89°	12-58	10-47	3-32	5-56	14.1	101°	13-53	11-36	4-11	7-02	17.5
90°	13-03	10-52	3-35	6-01	14.4	102°	13-57	11-40	4-14	7-08	17.8
91°	13-07	10-56	3-39	6-07	14.7	103°	14-02	11-44	4-17	7-13	18.1
92°	13-12	11-00	3-42	6-12	15.0						

DEEP FOCUS

Depths (*h*) measured from bottom of surface layers

	P-O					*P-O*			
h →	100 km.	300 km.	500 km.	700 km.	*h* →	100 km.	300 km.	500 km.	700 km.
Δ	*m s*	*m s*	*m s*	*m s*	Δ	*m s*	*m s*	*m s*	*m s*
0°	18	42	1-04	1-23	17°	3-51	3-39	3-29	3-24
1°	24	44	1-05	1-24	18°	4-03	3-49	3-39	3-33
2°	35	50	1-09	1-26	19°	4-14	3-59	3-48	3-42
3°	48	58	1-14	1-30	20°	4-24	4-09	3-58	3-51
4°	1-01	1-08	1-21	1-35	21°	4-34	4-19	4-07	4-00
5°	1-14	1-19	1-30	1-42	22°	4-44	4-28	4-16	4-09
6°	1-28	1-31	1-39	1-48	23°	4-54	4-38	4-25	4-18
7°	1-41	1-43	1-48	1-56	24°	5-03	4-47	4-34	4-26
8°	1-55	1-55	1-58	2-04	25°	5-13	4-56	4-43	4-35
9°	2-08	2-07	2-08	2-13	26°	5-22	5-05	4-52	4-44
10°	2-21	2-19	2-19	2-21	27°	5-31	5-14	5-01	4-52
11°	2-35	2-31	2-29	2-30	28°	5-40	5-23	5-09	5-01
12°	2-48	2-43	2-39	2-39	29°	5-49	5-32	5-18	5-09
13°	3-01	2-55	2-49	2-48	30°	5-58	5-41	5-27	5-17
14°	3-13	3-07	2-59	2-57	31°	6-07	5-49	5-35	5-26
15°	3-26	3-18	3-09	3-06	32°	6-16	5-58	5-44	5-34
16°	3-39	3-29	3-19	3-15	33°	6-24	6-06	5-52	5-42

DEEP FOCUS.—(*Continued*)

P-O					*P-O*				
$h \rightarrow$	100 km.	300 km.	500 km.	700 km.	$h \rightarrow$	100 km.	300 km.	500 km.	700 km.
Δ	*m s*	*m s*	*m s*	*m s*	Δ	*m s*	*m s*	*m s*	*m s*
34°	6-33	6-15	6-00	5-50	69°	10-53	10-32	10-13	9-58
35°	6-41	6-23	6-09	5-59	70°	10-59	10-38	10-19	10-04
36°	6-50	6-32	6-17	6-07	71°	11-05	10-44	10-25	10-10
37°	6-58	6-40	6-25	6-15	72°	11-11	10-50	10-31	10-16
38°	7-07	6-48	6-33	6-23	73°	11-17	10-55	10-37	10-21
39°	7-15	6-56	6-41	6-31	74°	11-23	11-01	10-42	10-27
40°	7-23	7-05	6-50	6-39	75°	11-28	11-07	10-48	10-32
41°	7-31	7-13	6-58	6-47	76°	11-34	11-13	10-53	10-38
42°	7-40	7-21	7-06	6-53	77°	11-40	11-18	10-59	10-43
43°	7-48	7-29	7-13	7-02	78°	11-45	11-23	11-04	10-48
44°	7-56	7-37	7-21	7-10	79°	11-51	11-29	11-10	10-53
45°	8-04	7-45	7-29	7-17	80°	11-56	11-34	11-15	10-59
46°	8-12	7-53	7-37	7-25	81°	12-01	11-39	11-20	11-04
47°	8-19	8-00	7-44	7-32	82°	12-06	11-44	11-25	11-09
48°	8-27	8-08	7-52	7-40	83°	12-12	11-50	11-30	11-13
49°	8-35	8-16	8-00	7-47	84°	12-17	11-55	11-35	11-18
50°	8-43	8-23	8-07	7-54	85°	12-22	11-59	11-40	11-23
51°	8-50	8-31	8-14	8-01	86°	12-26	12-04	11-45	11-28
52°	8-58	8-38	8-22	8-08	87°	12-31	12-09	11-49	11-33
53°	9-05	8-45	8-29	8-15	88°	12-36	12-14	11-54	11-37
54°	9-12	8-52	8-36	8-22	89°	12-41	12-19	11-59	11-42
55°	9-20	9-00	8-43	8-29	90°	12-46	12-23	12-03	11-46
56°	9-27	9-07	8-50	8-36	91°	12-50	12-28	12-08	11-51
57°	9-34	9-14	8-57	8-43	92°	12-55	12-33	12-13	11-56
58°	9-41	9-21	9-03	8-49	93°	12-59	12-37	12-17	12-00
59°	9-48	9-27	9-10	8-56	94°	13-04	12-42	12-22	12-05
60°	9-55	9-34	9-17	9-02	95°	13-09	12-46	12-26	12-09
61°	10-02	9-41	9-23	9-09	96°	13-13	12-51	12-31	12-14
62°	10-08	9-47	9-30	9-15	97°	13-18	12-55	12-35	12-18
63°	10-15	9-54	9-36	9-22	98°	13-22	13-00	12-40	12-23
64°	10-21	10-00	9-43	9-28	99°	13-27	13-04	12-44	12-27
65°	10-28	10-07	9-49	9-34	100°	13-31	13-09	12-49	12-32
66°	10-34	10-13	9-55	9-40	101°	13-36	13-13	12-53	12-36
67°	10-41	10-19	10-01	9-46	102°	13-40	13-18	12-58	12-41
68°	10-47	10-25	10-07	9-52	103°	13-45	13-22	13-02	

DEEP FOCUS, *S-P* INTERVAL
Depths (h) measured from bottom of surface layers

$h \rightarrow$	100 km.	300 km.	500 km.	700 km.	$h \rightarrow$	100 km.	300 km.	500 km.	700 km.
↓ Δ°	*m s*	*m s*	*m s*	*m s*	↓ Δ°	*m s*	*m s*	*m s*	*m s*
0°	14	33	50	1-06	37°	5-35	5-21	5-09	5-01
1°	18	35	51	1-07	38°	5-42	5-28	5-16	5-07
2°	26	39	54	1-09	39°	5-49	5-34	5-22	5-13
3°	36	46	59	1-12	40°	5-55	5-41	5-29	5-20
4°	47	54	1-04	1-16	41°	6-02	5-47	5-35	5-26
5°	57	1-02	1-11	1-21	42°	6-08	5-54	5-41	5-32
6°	1-08	1-12	1-18	1-26	43°	6-15	6-00	5-48	5-38
7°	1-19	1-21	1-26	1-32	44°	6-21	6-06	5-54	5-45
8°	1-29	1-30	1-34	1-39	45°	6-28	6-13	6-00	5-51
9°	1-40	1-40	1-42	1-46	46°	6-34	6-19	6-06	5-57
10°	1-51	1-50	1-51	1-53	47°	6-40	6-25	6-13	6-03
11°	2-01	1-59	1-59	2-00	48°	6-47	6-32	6-19	6-09
12°	2-12	2-09	2-08	2-07	49°	6-53	6-38	6-25	6-15
13°	2-22	2-19	2-16	2-15	50°	6-59	6-44	6-31	6-21
14°	2-32	2-29	2-25	2-22	51°	7-05	6-50	6-37	6-27
15°	2-43	2-38	2-33	2-29	52°	7-11	6-56	6-43	6-34
16°	2-53	2-48	2-41	2-36	53°	7-18	7-03	6-49	6-40
17°	3-03	2-56	2-49	2-44	54°	7-24	7-09	6-55	6-46
18°	3-13	3-05	2-56	2-51	55°	7-30	7-15	7-02	6-52
19°	3-23	3-13	3-04	2-58	56°	7-36	7-21	7-08	6-58
20°	3-32	3-21	3-11	3-05	57°	7-42	7-27	7-14	7-04
21°	3-41	3-29	3-19	3-12	58°	7-48	7-33	7-20	7-09
22°	3-49	3-36	3-26	3-19	59°	7-54	7-39	7-26	7-15
23°	3-57	3-44	3-33	3-26	60°	8-00	7-45	7-31	7-21
24°	4-04	3-51	3-40	3-33	61°	8-06	7-51	7-37	7-27
25°	4-12	3-58	3-47	3-40	62°	8-12	7-57	7-43	7-33
26°	4-19	4-05	3-54	3-47	63°	8-18	8-03	7-49	7-38
27°	4-26	4-12	4-01	3-54	64°	8-24	8-09	7-55	7-44
28°	4-33	4-19	4-07	4-00	65°	8-30	8-14	8-00	7-50
29°	4-40	4-26	4-14	4-07	66°	8-35	8-20	8-06	7-55
30°	4-47	4-33	4-21	4-14	67°	8-41	8-26	8-12	8-01
31°	4-54	4-40	4-28	4-21	68°	8-47	8-31	8-17	8-06
32°	5-01	4-47	4-35	4-28	69°	8-53	8-37	8-23	8-11
33°	5-08	4-54	4-42	4-34	70°	8-58	8-43	8-28	8-17
34°	5-14	5-00	4-49	4-41	71°	9-04	8-48	8-34	8-22
35°	5-21	5-07	4-56	4-48	72°	9-09	8-53	8-39	8-27
36°	5-28	5-14	5-02	4-54	73°	9-15	8-59	8-44	8-32

Deep Focus, *S-P* Interval.—(*Continued*)

$h \rightarrow$	100 km.	300 km.	500 km.	700 km.	$h \rightarrow$	100 km.	300 km.	500 km.	700 km.
$\downarrow \Delta°$	*m s*	*m s*	*m s*	*m s*	$\downarrow \Delta°$	*m s*	*m s*	*m s*	*m s*
74°	9-20	9-04	8-49	8-37	89°	10-35	10-17	10-02	9-49
75°	9-25	9-09	8-55	8-43	90°	10-39	10-22	10-06	9-53
76°	9-31	9-14	9-00	8-48	91°	10-43	10-26	10-10	9-57
77°	9-36	9-19	9-05	8-53	92°	10-48	10-30	10-14	10-01
78°	9-41	9-25	9-10	8-58	93°	10-52	10-34	10-18	10-05
79°	9-46	9-30	9-15	9-03	94°	10-56	10-38	10-22	10-09
80°	9-51	9-35	9-20	9-08	95°	11-00	10-42	10-26	10-12
81°	9-56	9-40	9-25	9-13	96°	11-04	10-46	10-30	10-16
82°	10-01	9-45	9-30	9-17	97°	11-08	10-50	10-34	10-20
83°	10-06	9-50	9-35	9-22	98°	11-11	10-54	10-38	10-24
84°	10-11	9-54	9-39	9-27	99°	11-15	10-57	10-42	10-28
85°	10-16	9-59	9-44	9-31	100°	11-19	11-01	10-46	10-32
86°	10-21	10-04	9-49	9-36	101°	11-23	11-05	10-50	10-36
87°	10-26	10-08	9-53	9-40	102°	11-27	11-09	10-53	10-40
88°	10-31	10-13	9-58	9-44	103°	11-31	11-13	10-57	

Deep Focus, *pP-P* Interval

h	100 km.	200 km.	300 km.	400 km.	500 km.	600 km.	700 km.
Δ	*m s*	*m s*	*m s*	*m s*	*m s*	*m s*	*m s*
22°	26	42					
25°	27	44	59	1-12			
30°	29	47	1-03	1-17	1-29		
35°	29	48	1-05	1-20	1-33	1-42	
40°	30	49	1-06	1-23	1-36	1-47	
45°	30	50	1-08	1-24	1-38	1-50	1-59
50°	30	51	1-09	1-26	1-41	1-53	2-04
55°	31	52	1-11	1-28	1-44	1-57	2-09
60°	32	53	1-13	1-31	1-46	2-01	2-14
65°	32	54	1-14	1-32	1-49	2-04	2-18
70°	33	55	1-15	1-34	1-51	2-07	2-21
75°	33	55	1-16	1-35	1-53	2-09	2-24
80°	34	56	1-17	1-37	1-55	2-11	2-27
85°	34	56	1-18	1-38	1-56	2-13	2-29
90°	35	57	1-18	1-39	1-58	2-15	2-32
95°	35	57	1-19	1-39	1-58	2-16	2-32
100°	35	57	1-19	1-39	1-58	2-16	2-33

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CHAPTER XI
Location of Epicenters

Introduction

The first problem facing the seismographer with the records of an earthquake before him is the location of the epicenter. If he has the records from several stations, or the readings, he may apply one of two general methods. The first makes use of the arrival times at each station of one group of waves, in practice the P group. It is necessary that the clock correction be known accurately at each station. At present the importance of accurate time is recognized at most stations, so that this method is quite practicable. The second general method employs the time interval between the arrival of two groups, in practice usually S and P. In this method it is not necessary to know the clock correction at each station used. It is only necessary that the clock at a given station does not gain or lose an appreciable time in the short interval between P and S. However, this interval method requires the identification of the beginning of S in addition to that of P. Now it is generally a simpler matter to find the beginning of the first motion on a record than it is to identify the beginning of a later group on a record already complicated by the vibrations of earlier groups. Also S is notorious for its multiplicity; in many ranges it apparently consists of more than one group. For instance, it frequently begins with small motion, sometimes called the "curtsey," followed by a sharp increase to large motion. If the epicenter is to be located by the use of existing tables connecting the S-P interval with epicentral distance, there is the additional uncertainty of the dependability of the S curve on which the tables were based.

Accurate location of epicenters is therefore based on the arrival of P. In the case of deep-focus earthquakes the arrival time of one of the waves reflected near the epicenter is also needed to fix the depth. The S-P interval, as well as the travel times of P and S, varies with depth of focus. In using the P times to get the epicenter the method may be either (a) to locate an epicenter such that the P times fit one of the standard P travel time tables, or (b) to locate the epicenter so that the P arrival times fit a smooth curve, in practice one which is linear in certain ranges.

Travel time curves

An accurate and complete set of travel time tables is necessary equipment for a seismologist. At the time of writing the most recent available are: *Seismological Tables* by Harold Jeffreys and K. E. Bullen, British Association for the Advancement of Science, Gray-Milne Trust, London, 1940. The Rev. J. B. Macelwane, S.J., of St. Louis University has in the past issued a number of mimeographed travel time tables and a chart for deep-focus earthquakes (by G. J. Brunner, S.J.). Gutenberg and Richter have from time to time published tables of travel times.

Geiger method[1]

In order to fix the epicenter so that the travel times best fit a standard curve, one first locates an approximate epicenter. This may be done, for instance, by measuring the S-P intervals at several stations, looking up in the tables the corresponding distances, and locating on a large terrestrial globe the epicenter best fitting these distances. The latitude and longitude of this approximate epicenter may be called φ^* and λ^*.

The epicentral distance to each station from the assumed epicenter $\varphi^* \lambda^*$ is then computed. Then the corresponding travel time read from the tables is subtracted from the arrival time of P at that station. The remainder is the time

of occurrence as computed from that station. The mean of these times of occurrence as computed from all the stations is then taken as t_0^*, the assumed time of occurrence.

Let t_n^* be the computed arrival time at each station based on the assumed epicenter and time of occurrence; t_n is the observed arrival time.

Now if our discrepancies $F_n = t_n - t_n^*$ are small, we may write

$$\begin{aligned} t_n &= t_n^* + F_n \\ &= t_n^* + \frac{\partial t_r^*}{\partial \lambda_0^*} d\lambda_0^* + \frac{\partial t_n^*}{\partial \varphi_0^*} d\varphi_0^* + \frac{\partial t_n^*}{\partial t_0^*} dt_0^*. \end{aligned}$$

We may evaluate

$$\frac{\partial t_n^*}{\partial \lambda_0^*} \quad \text{and} \quad \frac{\partial t_n^*}{\partial \varphi_0^*}$$

at each station by shifting the epicenter slightly, computing the corresponding change in epicentral distance, and finding from the tables the corresponding change in travel time of P and therefore of t_n^*. We then have n equations with unknown $d\lambda_0^*$, $d\varphi_0^*$, and dt_0^*. These may be solved by the method of normal equations (least squares). The final epicentral location is then at

$$\begin{aligned} \lambda_0 &= \lambda_0^* + d\lambda_0^*, \\ \varphi_0 &= \varphi_0^* + d\varphi_0^*, \\ t_0 &= t_0^* + dt_0^*. \end{aligned}$$

The method as applied to epicenters was first used by Geiger.

The computation of epicentral distances may be made by use of the formula of spherical trigonometry

$$\cos \Delta = \cos e \cos s + \sin e \sin s \cos P,$$

where Δ is the epicentral distance, e is the colatitude of the station, 90° ± latitude of station (+ if south), s is the colatitude of the epicenter, 90° ± latitude of epicenter (+ if south), and P is the polar angle between the meridians of station and epicenter. From the relationship it also

follows that

$$\frac{\partial \Delta}{\partial \varphi_0} = -\cos E,$$

where φ_0 is the latitude of the epicenter and E is the bearing of the station at the epicenter (taken as a positive angle between 0° and 180°). Also

$$\pm \frac{\partial \Delta}{\partial \lambda_0} = \frac{\partial \Delta}{\partial P} = \cos \varphi_0 \sin E.$$

The sign of this term depends on the relative longitudes of station and epicenter, that is, on whether Δ is increased by an increase of λ_0, $\partial\Delta/\partial P$ being inherently positive.

The last two equations may be used in computing the differential coefficients in the least square adjustment if E has been computed.

The following three equations are useful in determining Δ as well as E and S (the bearing of the epicenter at the station):

$$\log \tan \frac{E+S}{2} = \log \cot \frac{P}{2} + \log \cos \frac{e-s}{2} + \operatorname{colog} \cos \frac{e+s}{2}.$$

$$\log \tan \frac{E-S}{2} = \log \cot \frac{P}{2} + \log \sin \frac{e-s}{2} + \operatorname{colog} \sin \frac{e+s}{2}.$$

$$\log \tan \frac{\Delta}{2} = \log \sin \frac{E+S}{2} + \operatorname{colog} \sin \frac{E-S}{2} + \log \tan \frac{e-s}{2}.$$

Geocentric latitudes should be used in the computations.[2]

Straight line method

For certain ranges of epicentral distance the travel time curve of P is a straight line within our limits of accuracy in measuring time. For example, from about 5° to about 18° (possibly 20°) this holds true. For earthquakes recorded close to the source the travel time of $\overline{P}$ (P traveling all of its path in the granitic layer) is sensibly a straight line as soon as the distance from the epicenter becomes large compared to the depth of focus. The equation of

the travel time curve in these regions is then

$$t = \frac{\Delta}{v} + I,$$

where v is the apparent surface speed and I is the time intercept of the branch of the curve under consideration.

We want to find an epicenter and value of I such that for each station

$$t_n - \left(\frac{\Delta_n}{v} + I\right) = 0.$$

Assume an epicenter, a value of v, and a value of I as accurately as approximate methods allow (by *S-P* intervals, the use of field data, and previous knowledge of the speed). Let the latitude and longitude of the assumed epicenter be $\varphi_0{}^*\lambda_0{}^*$, the assumed I be I^*, and the assumed speed v^*. The computed distance to this assumed epicenter from any station is $\Delta_n{}^*$. In general the substitution of $\Delta_n{}^*$ and I^* in the last equation will not satisfy it for all stations. We wish to find corrections to $\varphi_0{}^*$, $\lambda_0{}^*$, $I_0{}^*$, such that when the corrected values are substituted in the equations they will be satisfied in the best possible fashion; that is, the sum of the squares of the discrepancies will be a minimum. If the values assumed are near the best values so that the corrections required are only small, we may write

$$(t_n - I^* - dI)(v^* + dv) - \frac{\partial \Delta}{\partial \varphi_0} d\varphi_0 - \frac{\partial \Delta}{\partial \lambda_0} d\lambda_0 = \Delta_n{}^*,$$

or to first orders in small quantities,

$$(t_n - I^*)dv - v^*dI - \frac{\partial \Delta}{\partial \varphi_0} d\varphi_0 - \frac{\partial \Delta}{\partial \lambda_0} d\lambda_0 = \Delta_n{}^* - (t_n - I^*)v^*.$$

The unknowns are dv, dI, $d\varphi_0$, $d\lambda_0$, which are corrections to be added to the assumed values. If there are more than four equations of this type, they may be solved by the method of normal equations to get the best values. The theoretical background of the use of least squares methods

on sparse data may be questionable, but it is one method of adjustment generally agreed upon and gives the uniformity of method desirable if we are to compare our results.

The station pair method

If we can find two stations at which *P* arrives at the same time, we can say that the epicenter lies on a great circle which bisects the great circle joining the stations and is normal to it at the intersection. This conclusion involves the assumption of equal speeds in all directions which is resident in all the methods. Experience indicates it comparatively sound in earthquake seismology. Doubtless greater refinements in the accuracy of measuring time will eventually bring in small differences due to local geology near focus and station.

If two such pairs of stations are available, the epicenter is at one of the two intersections of the two great circles so constructed. Usually these are far enough apart so that the other features of the seismograms (such as *S-P* interval, for instance) indicate which is the correct one. More station pairs improve the method. Since exact coincidence of time of arrival at two stations is infrequent, it is usual to take two stations at which the arrival time of *P* is nearly the same. Since the slope of the *P* travel time curve varies slowly and the general region of the epicenter is usually known, one may obtain the approximate value of the apparent surface speed in the regions of the stations from the travel time curve, multiply it by the difference in arrival time at the two stations, and obtain the difference in distance to the epicenter of the two stations. This gives a curve on the earth's surface which is very near a great circle and may be drawn on a globe or map.

Interval method

The method most commonly used for quick determination of epicenters makes use of the time interval between the arrivals of *P* and *S*. This interval is a function of

epicentral distance and is tabulated in many tables. The epicentral distance from three stations determines the epicenter. In identifying P and S one should always check the later interval between S and the surface waves. If this interval fails to check the tables by several minutes (a close check is not to be expected as the arrival times of surface waves show a considerable scatter), it is probable that the groups called P and S have not been properly identified. For example, the records from the Bosch-Omori seismographs at Berkeley from Indian earthquakes sometimes show PP as the first wave and later SKS, the general appearance of these phases being that of P and S for shocks

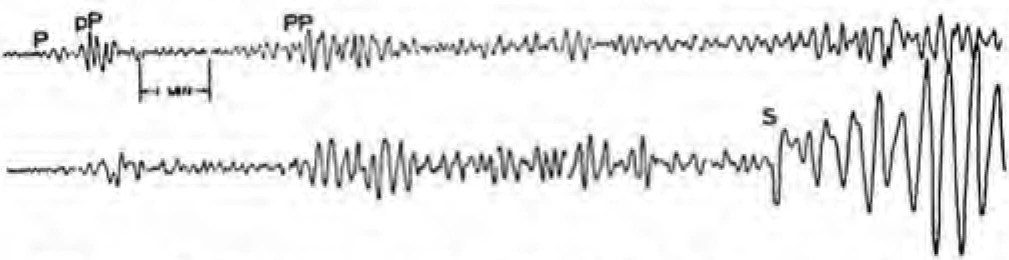

Fig. 51. Seismograms Recorded at Berkeley of the Rumanian Earthquake of November 9, 1940, Epicentral Distance about 98°, Upper Record Vertical Component Lower Record North-South Component.

of less epicentral distance. However, the delay of the surface waves after SKS indicates clearly that the epicenter is very distant.

Intervals between other groups of waves (for example, PP and SKS) may be used to get epicentral distances provided those waves can be identified. However, it is almost always S-P which is so used. In early seismology the arrival times of the surface waves were used, but the great scatter of these times renders them no longer available for this purpose now when greater accuracy is possible. If the earthquake is of deep focus pP, the longitudinal wave reflected near the epicenter may be identified (see Figure 51). The interval $pP - P$ is not very sensitive to variation of epicentral distance. One may take first the epicentral distance roughly from S-P and travel time curves for normal depth. For this distance he finds what depth of focus

corresponds to $pP - P$. Then he consults the S-P curves for that depth to fix accurately the epicentral distance.

Once the epicentral distances are obtained from several stations, it is necessary to use a globe or a projection to find the intersection of the circles around the stations corresponding to these distances. If a large globe is not available, the stereographic projection is convenient and accurate. (Of course for small shocks recorded at near stations almost any map will do, curvature of the earth not entering into the problem appreciably.) First the primitive circle is

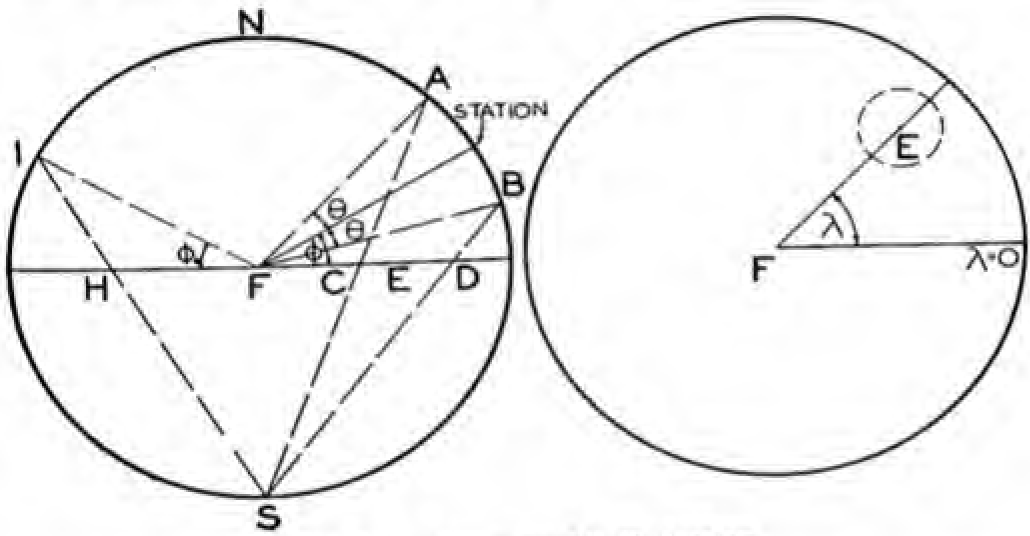

Fig. 52. Stereographic Projection.

drawn representing a great circle of the earth through the poles and a station. The trace of the equatorial plane is a diameter of this circle. The station is plotted on this circle according to its latitude, φ (see Figure 52). The angular epicentral distance, θ, is then laid off on either side of the station point. The two points, A and B, so obtained are traces of a circle on the globe which cuts our primitive circle at these points. This circle on the imagined globe is to be projected on the equatorial plane by joining A and B to the pole S. By the property of the stereographic projection a circle on the globe projects a circle on the equatorial plane. The circle projected on the equatorial plane by

circle AB on the globe has a diameter CD. Its center E (halfway between C and D) is distance FE from the center of the equatorial plane.

Next the equatorial circle of same diameter as the primitive circle is constructed (Figure 52), and the radius corresponding to the longitude, λ, of the station is drawn. On this radius, FE is laid off as measured from the primitive circle. About it is drawn a circle of diameter CD. We have now projected the circle about the station on the globe onto the equatorial plane of the globe. This is done for all the stations for which data are available. These circles on the equatorial plane will intersect more or less closely at a point. The best intersection is picked; its longitude λ_0 is read directly from the equatorial circle. Its distance from F, say FH, is measured and plotted back on the primitive circle, giving point H. This point is projected back onto the sphere at I. I is connected to F, and the latitude of the epicenter, φ_0, is read.

Method of direction

The first group of waves to record at a station are longitudinal in character. The direction of vibration of the particles along the path below the earth's surface is back and forth in the direction of the path. As the wave arrives at the surface, it generates reflected waves and the surface motion is warped from the direction of the path. All motion in both incident and reflected waves contributes to the surface motion so that the apparent angle of incidence at the surface as computed from the recorded horizontal amplitude $\bar{u}_3$ and vertical $\bar{u}_1$ by

$$\tan \bar{\alpha} = \frac{\bar{u}_3}{\bar{u}_1}$$

does not give the direction of approach of the wave. This was shown in chapter 9 (see equation 63). However, the horizontal amplitude $\bar{u}_3$ does lie in the great circle joining epicenter and station, and its direction may be obtained by compounding as vectors the amplitudes of the first wave of

P on the *E-W* and *N-S* component seismograms. The great circle once located, the question arises as to which way along it the epicenter lies. This information is obtained from the vertical component of the first motion. If it is up, the first wave was a compression and the horizontal motion was away from the epicenter. If, on the other hand, the first vertical motion was down, the wave was a rarefaction and the horizontal movement was toward the epicenter. It is important that the three component motions compounded arrive at the same time. If the beginning of *P* is poor on one component, a slightly later wave which appears well recorded and arrives at the same time on all components may be used. Of course it must be a wave of the *P* group. The direction of the epicenter having been so obtained, the distance may be computed from the *S-P* interval at the station, and thus an approximate location of the epicenter obtained from the records of one station.

It is to be noted that the direct trace amplitudes, those amplitudes measured direct from the records, may not be used for quantitative determination of the azimuth of the epicenter unless the constants of the two horizontal component instruments are very nearly the same. Otherwise earth amplitudes must be computed. Again, if the free periods of the components vary widely—for example, if the vertical seismograph has a period very short compared to the horizontals—it may be that the different components will record waves of different period, many being present in the first motion. It has not yet been established whether or not all frequencies in a given shock begin with the same phase of motion, that is, with compression or rarefaction.

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CHAPTER XII

Seismograms

Integration

The seismograph writes a record of the differential motion between its mass and the pier on which it sits. A major problem of seismology is the determination of earth motion—the motion of the pier relative to earth as a whole—from this record. The equation of the simple seismograph, equation 2, from Chapter VIII, is

$$\frac{d^2x}{dt^2} + 2\kappa\frac{dx}{dt} + n^2x = -V\frac{d^2y}{dt^2}, \tag{88}$$

where x is trace amplitude on the seismogram, y is the earth amplitude, κ a damping coefficient, $n = 2\pi/T_0$ [where T_0 is the free period (undamped) of the seismograph pendulum], and V is the static magnification. This equation assumes that the displacements of the pendulum are small, which is reasonable, at least for high magnifications. It assumes that all damping is proportional to the speed, dx/dt. The assumption can be no more than an approximation, but, if solid friction is largely removed, the results of shaking-table investigations, in which table motion is compared with the motion obtained by integrating the record by use of the equation, indicate that the errors introduced are not gross.

Equation 88 further assumes that displacements of the pendulum due to rotation of its support (tilting) are negligible compared to those due to translations. Angenheister has pointed out that this is so in the practical earthquake case[1].

Let the x axis be taken along the earth's surface and the z axis vertical down. Let a plane wave be incident at an angle α in the xz plane with vibration in that plane. Assume simple harmonic motion. Let u be the wave displacement in the horizontal and w that in the vertical; then

$$w = W \sin\left(\frac{2\pi t}{T} + \delta_1\right) = W \sin\left(\frac{2\pi x \sin\alpha}{\lambda} + \delta_1\right),$$

where T is the period and λ the wave length. The tangent of the angle of tilt, ψ, is

$$\tan\psi = \frac{dw}{dx} = \frac{2\pi W \sin\alpha}{\lambda} \cos\left(\frac{2\pi x \sin\alpha}{\lambda} + \delta_1\right).$$

In a distant earthquake λ may vary from, say 8 kilometers for short P waves of period 1 second to, say, 140 kilometers for long M waves of period 40 seconds (the G waves will produce no tilt). W varies from a few microns to, say 2 millimeters for long waves. The maximum amplitude of $\tan\psi$ will be of order of about 10^{-7}.

For large local shocks we take the case of the strong motion records in the Long Beach earthquake[2].

$$a_x = 108\,\frac{\text{cm.}}{\text{sec}^2}, \qquad a_z = 69\,\frac{\text{cm.}}{\text{sec}^2},$$
$$T = 1.3 \text{ sec.}, \qquad W = 3 \text{ cm.}$$

Take

$$v = 3\,\frac{\text{km}}{\text{sec.}},$$

so

$$\lambda = 39 \times 10^4 \text{ cm.}$$

The amplitude of $\tan\psi$ is then, say, 10^{-4} to 10^{-5}.

The horizontal acceleration of the earthquake, combined vectorially with the vertical acceleration plus gravity, produces an apparent tilt ψ_a, where

$$\tan\psi_a = \frac{a_x}{g + a_z}.$$

For the Long Beach example above cited both a_x and a_z approximate 0.1 g, so

$$(\tan \psi_a)_{\text{max.}} \doteq 10^{-1}.$$

For distant shocks (U is maximum horizontal displacement)

$$(\tan \psi_a)_{\text{max.}} \doteq \frac{\left(\frac{2\pi}{T}\right)^2 U}{g},$$

and $\tan \psi_a/\tan \psi$ is at least of order of $10^{-2}\lambda/T^2$.

So for distant shocks apparent tilt is likely to be at least 10^2 times the true tilt and for strong local shocks 10^3 times.

Neglect of true tilt seems, therefore, physically allowable as well as mathematically necessary.

The determination of the correct value of n for use in equation 88 may be a problem, since in many instruments with electromagnetic damping the addition of the magnets after the free period is measured introduces an extra restoring force as well as a damping force. This addition may be due to magnetic impurities in the damping vane or to tilting of the seismograph base by the added load. Thus a free period measured with damping removed is not the effective free period with damping restored.

Integrating equation 88, we have

$$\frac{dx}{dt} - \left(\frac{dx}{dt}\right)_{t=0} + 2\kappa(x - x_0) + n^2 \int_0^t x\,dt = -V\left\{\frac{dy}{dt} - \left(\frac{dy}{dt}\right)_{t=0}\right\}.$$

Integrating again, we get

$$y = -\frac{x}{V} - \frac{2\kappa}{V}\int_0^t x\,dt - \frac{n^2}{V}\int_0^t dt \int_0^t x\,dt$$
$$+ \frac{x_0}{V} + y_0 + \frac{1}{V}\left\{V\left(\frac{dy}{dt}\right)_{t=0} + \left(\frac{dx}{dt}\right)_{t=0} + 2\kappa x_0\right\} t. \quad (89)$$

If it is possible to begin the integration at a time when the pendulum is at rest relative to its pier (just before the earthquake when no microseisms are present), the terms

after the double integral vanish by virtue of the vanishing of initial displacements and velocities. If integration is begun at the exact moment that the earthquake arrives (no microseisms), $x_0 = 0$ and

$$V\left(\frac{dy}{dt}\right)_{t=0} = -\left(\frac{dx}{dt}\right)_{t=0},$$

so the terms again vanish.

Therefore, except in special cases, there is some difficulty with initial conditions. There is also difficulty in the measurement of the trace displacements x, since the exact determination of the rest position, from which they are measured, is difficult. If an arbitrary rest line is drawn, its equation may be written

$$x_0' = \alpha + \beta t.$$

The measured values of x, x', will then be

$$x' = x - \alpha - \beta t.$$

If $x - \alpha - \beta t$ is substituted for x in equation 88, integration as above leads to a modification in equation 89 of the constant terms, the coefficient of t, and the addition of terms in t^2 and t^3.

The double integral in equation 89 may be integrated by parts:

$$\int_0^t \left\{\int_0^t x dt\right\} dt = t\int_0^t x dt - \int_0^t t x dt.$$

If this is substituted in 89, there are two single integrals to be evaluated, and correction for an erroneous zero line will involve no term in t^3.

In general, integration of records leads to a fairly accurate curve of earth displacements superposed on a steady drift from the position of rest. This drift is usually removed arbitrarily. It is no doubt due either to incorrect choice of the rest line on the seismogram or to difficulties in determining initial conditions. Numerical integration of 89 is tedious. An integraph reduces the labor. In equation 89

if the period of the seismograph (T_0) is long (compared to the periods of the waves recorded), the terms multiplied by $\kappa \left(\propto \frac{1}{T_0}\right)$ and $n^2 \left(\propto \frac{1}{T_0^2}\right)$ are negligible compared to $\frac{x}{V}$. The earth displacement is proportional to the trace displacement. As T_0 becomes smaller, the first integral becomes important. Finally, if T_0 is very small (compared to periods recorded), the double integral becomes the important contributor to the right-hand member. This case is for the accelerometer.

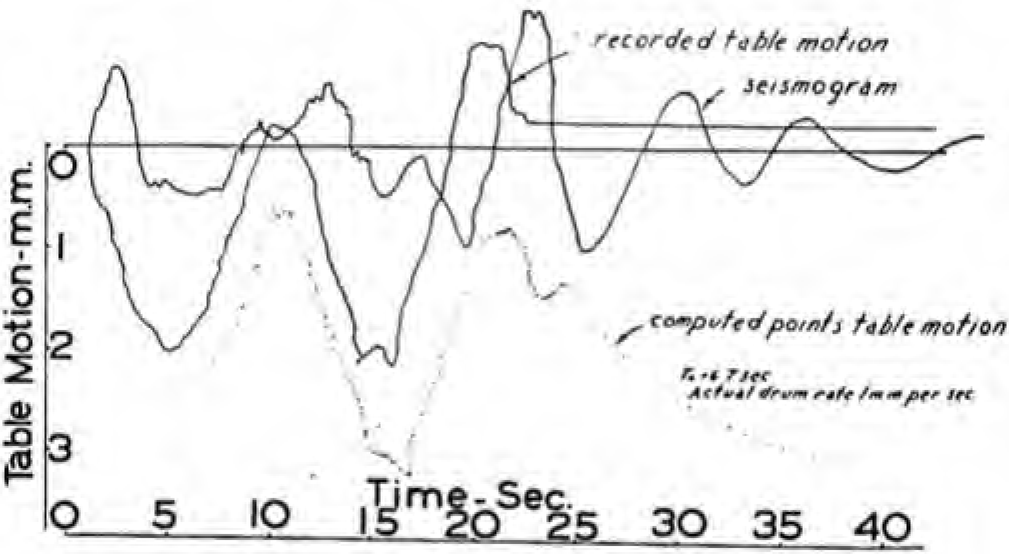

Fig. 53. Record from Shaking Table.

Figure 53, after Dyk[3], shows the results of integrating a record written by a seismograph on a shaking table. Not enough integration of seismograms has been done in seismological studies. Extensive use of it will doubtless lead to a greatly increased knowledge of the behavior of seismic waves.

The integration of the seismogram from a seismograph which employs a galvanometer in its registration is more complicated, involving a triple integration.

From equation 29 for an electromagnetic seismograph, we have

$$p^4\theta + ap^3\theta + bp^2\theta + cp\theta + m\theta = -Fp^3y,$$

where

$$\begin{aligned} a &= 2(\kappa + \kappa_1), \\ b &= 4\kappa\kappa_1(1 - \sigma^2) + n^2 + n_1^2, \\ c &= 2(\kappa n_1^2 + \kappa_1 n^2), \\ m &= n^2 n_1^2 \end{aligned}$$

and

$$F = \frac{hM}{K} k_g.$$

Integrating and inserting initial boundary values, we obtain

$$\begin{aligned} p^3\theta + ap^2\theta + bp\theta + c\theta + m \int_0^t \theta dt + Fp^2y \\ = (p^3\theta)_0 + a(p^2\theta)_0 + b(p\theta)_0 + c\theta_0 + F(p^2y)_0, \end{aligned}$$

where the zero subscripts indicate values at $t = 0$.

Now put

$$2B = (p^3\theta)_0 + a(p^2\theta)_0 + b(p\theta)_0 + c\theta_0 + F(p^2y)_0.$$

Integrate again, getting

$$p^2\theta + ap\theta + b\theta + c \int_0^t \theta dt + m \int_0^t dt \int_0^t \theta dt + Fpy = E + 2Bt,$$

where

$$E = (p^2\theta)_0 + a(p\theta)_0 + b\theta_0 + F(py)_0.$$

Integrate again, to get

$$\begin{aligned} p\theta + a\theta + b \int_0^t \theta dt + c \int_0^t dt \int_0^t \theta dt + m \int_0^t dt \int_0^t dt \int_0^t \theta dt \\ + F(y - y_0) = A + Et + Bt^2, \end{aligned}$$

in which

$$A = (p\theta)_0 + a\theta_0.$$

Now

$$x = L\theta,$$

where x is trace amplitude on the seismogram and L is twice the optical lever arm of the galvanometer recording system.

Therefore

$$y - y_0 = -\frac{1}{FL}\left\{px + ax + b\int_0^t xdt + c\int_0^t dt\int_0^t xdt \right.$$
$$\left. + m\int_0^t dt\int_0^t dt\int_0^t xdt - LA - LEt - LBt^2\right\}.$$

The values of A, E, and B are determined by the initial conditions and the instrumental constants.

If an erroneous choice of the zero line is made in measuring x, that is, if we choose the straight line $\theta = \alpha + \beta t$ for our reference instead of $\theta = 0$, then in equation 29, θ must be replaced by $\theta - \alpha - \beta t$, and when the integrations are accomplished the coefficients A, E, and B are altered and there are additional terms in t^3 and t^4. Errors in choice of initial conditions also contribute to the polynomial in time and are therefore cumulative.

The problem of integration of seismograms is treated in detail by Prince Galitzin in his *Vorlesungen über Seismometrie.*

Studies

Detailed studies of seismograms may be classified in two categories, the study of the travel times and the study of amplitudes and periods.

Travel times

Investigations of travel times lead to the construction of travel time curves or tables. The integration of such curves leads to conclusions regarding the variation of the speed of seismic waves at depth in earth as described in an earlier chapter. Providing the travel time curve can be well delineated, the method is good. There is grave danger that discontinuities in slope of the curve, with perhaps the existence of cusps, be lost in the construction of the curves. If only one earthquake is studied, the sparseness of data may well lead to such oversight. If the data from many shocks are included in the curve, the errors of different

magnitude and sign in different shocks lead to a scattering of points on the travel time curve which is likely to mask successfully any such discontinuity in slope unless it is very marked. An example is furnished by recent work of Gutenberg and Richter. In order to explain the prolongation of the P' curve back into the shadow zone after the fashion suggested by Miss Lehman, namely by sharp increases of the speed of P in the core, they have constructed a core which explains the early observation of P' but which requires cusps on its travel time curve not yet observed.

The travel time curves of P and S have led to conclusions regarding discontinuities, for example, the core boundary. The existence of the core boundary has been checked by the observation of P_cP and S_cS waves reflected at this boundary. Other reflections on the inner side of the boundary have been identified and the existence of the core further verified. Discontinuities above the core have not yet been checked by the identification of waves reflected by them. Perhaps this is because they are regions of gradual change taking place within a layer which is not thin when compared with the wave length of the seismic waves available for its study. Perhaps it is because sufficient efforts have not been directed toward identification of such waves. For the discontinuities in the earth's crust it is rarely if ever that stations have been sufficiently closely spaced around an epicenter for the observation of such reflected waves to be expected in sufficient number for the construction of a travel time curve. For a wave must be traced from station to station and its travel time curve constructed in order to establish its path.

Amplitudes

First motion. Some of the most productive studies of the amplitudes of waves have been those of the amplitude of first motion of P. An earthquake source sends out a first compression wave in some directions from the focus

and a first rarefaction wave in others. These first waves are followed by oscillatory motion. But the first wave gives a picture of the first release at the focus and may be used to determine the distribution of stress near the focus just before the shock. If this statement appears to need theoretical justification, it may be found in a paper of Nakano[4], and in others following it.

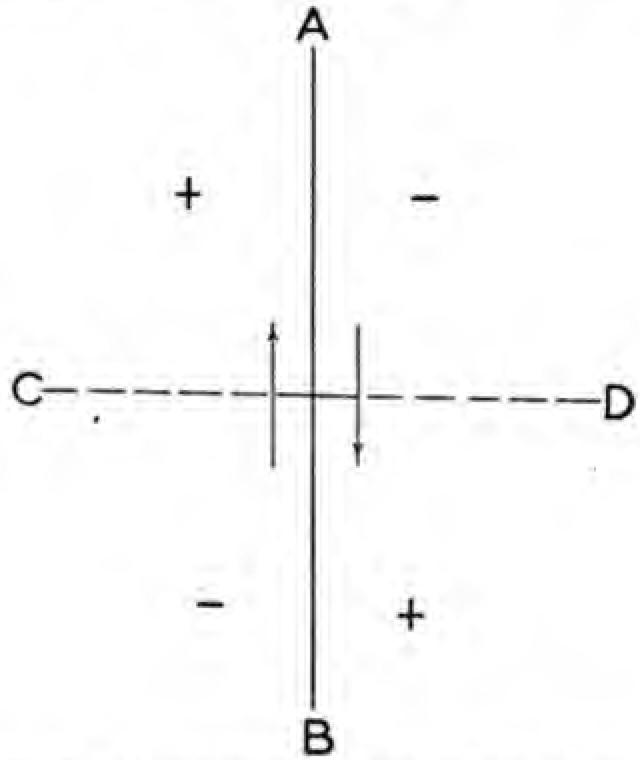

Fig. 54. Quadrant Distribution of First Motion.

A simple example of the relation between stress at focus and first motion of P is the case of a vertical fault caused by purely horizontal strain. In Figure 54, AB represents the trace of such a fault. The arrows represent the direction of motion of the two sides at the time of the earthquake. Sudden slippage such as this will send out first waves of compression in the second and fourth quadrants, marked + in the figure, and waves of first rarefaction in the first and third quadrants, marked − in the figure. Thus the earth would be divided into four quarters by the fault plane and a plane perpendicular to it. Stations in

the surfaces of any of these quarters would receive first $+P$ or $-P$ waves, according to the quarter. Complete coverage of all points on the earth's surface by observatories indicating such a quarter distribution would lead at once to the location of the two planes whose traces at the epicenter are AB and CD. Which of these was the fault is not determined by the method. If the more likely one may be picked by geologic field data, the direction of motion along it follows from the distribution of rarefactions and

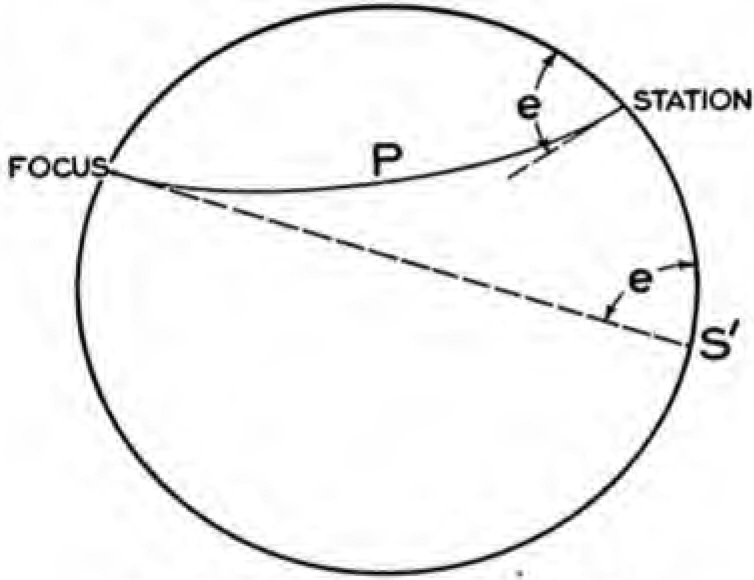

Fig. 55. Extended Position of Station.

compressions. A careful study of the amplitudes of S may also lead to a distinction of the two planes.

It is verified by observation that the nature of the first P wave does not vary from compression to rarefaction or *vice versa* as it progresses.

In general, of course, faults are not vertical and motion on them is not purely horizontal. Again the source of all earthquakes may not be due to couples acting along fault planes. However, the distribution of first motion in many large earthquakes may be explained as due to a couple acting along a fault.

In attempting such an analysis of first motions a stereographic projection is useful. Before projecting the position of the station on the equatorial plane it is necessary to plot its "extended" position, S', as shown in Figure 55. This is done by drawing chord FS' such that at F it is coincident with ray P; that is, it is the position on the earth's surface at which the ray leaving the focus along path P would have emerged if the path had not curved. Chord FS' is drawn by making angle e equal to the value of the angle of emergence at the station. A very deep focus would render this an assumption, but the angle of emergence is not known very accurately in any case. Values of this angle may be computed from a travel time curve, but an assumption as to the speed "near the surface" must be made.

The problem is to cut the earth by two perpendicular planes such that they divide the surface into four areas which show alternating compressions and rarefactions. One of these planes contains the fault plane across which the couple acts. The other plane bisects the arrows representing the couple; that is, it divides the region on each side of the fault in two parts such that on one side compression was first sent out and on the other rarefaction. It is the extended position of the station with which its + or − observation must be associated. These extended positions are projected stereographically, using the epicenter as the pole. Since circles on the globe project circles on the plane, we may seek to draw two circles on the projection dividing the + and − areas. If the depth of focus is negligible, these circles will both pass through the epicenter. If the fault is very nearly vertical, the two circles will be very nearly normal at the epicenter. It is to be remembered that in the stereographic projection the angle between two curves is preserved. The more complete the observation of the first motions—the more points on the earth's surface at which the first motion has been observed—the better the delineation of these circles. Figure 56 shows such circles drawn for the earthquake of July 6, 1934, off the

coast of northern California. Circle AA, of diameter R, is the projection of the fault plane. (It is shown as fault plane, rather than BB, on geological grounds.) Figure 57 (not drawn to scale) indicates how the azimuth and dip of the plane are obtained from this circle on the equatorial plane and from the polar plane. The dip corresponding to Figure 56 is large (84°); therefore the secondary circle

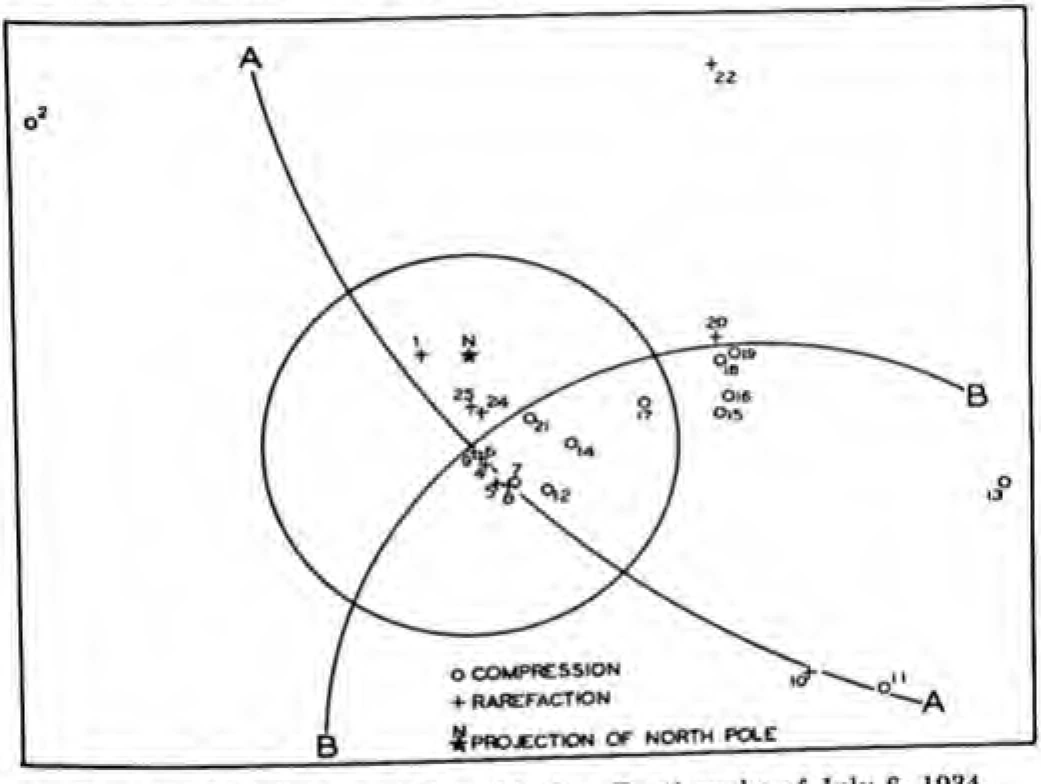

Fig. 56. Distribution of First Motion, Earthquake of July 6, 1934.

may be constructed nearly normal to AA in Figure 56. (Otherwise the data do not delineate it clearly.) This secondary circle gives the azimuth and dip of the plane at right angles to the couple acting along the fault. For the case of Figure 56, the couple indicated was such as to move the Pacific side northerly and down relative to the continental side, the motion dipping down toward the north by some 26°. Movement of the Pacific side northerly relative to the landward side is the present habit of California faulting.

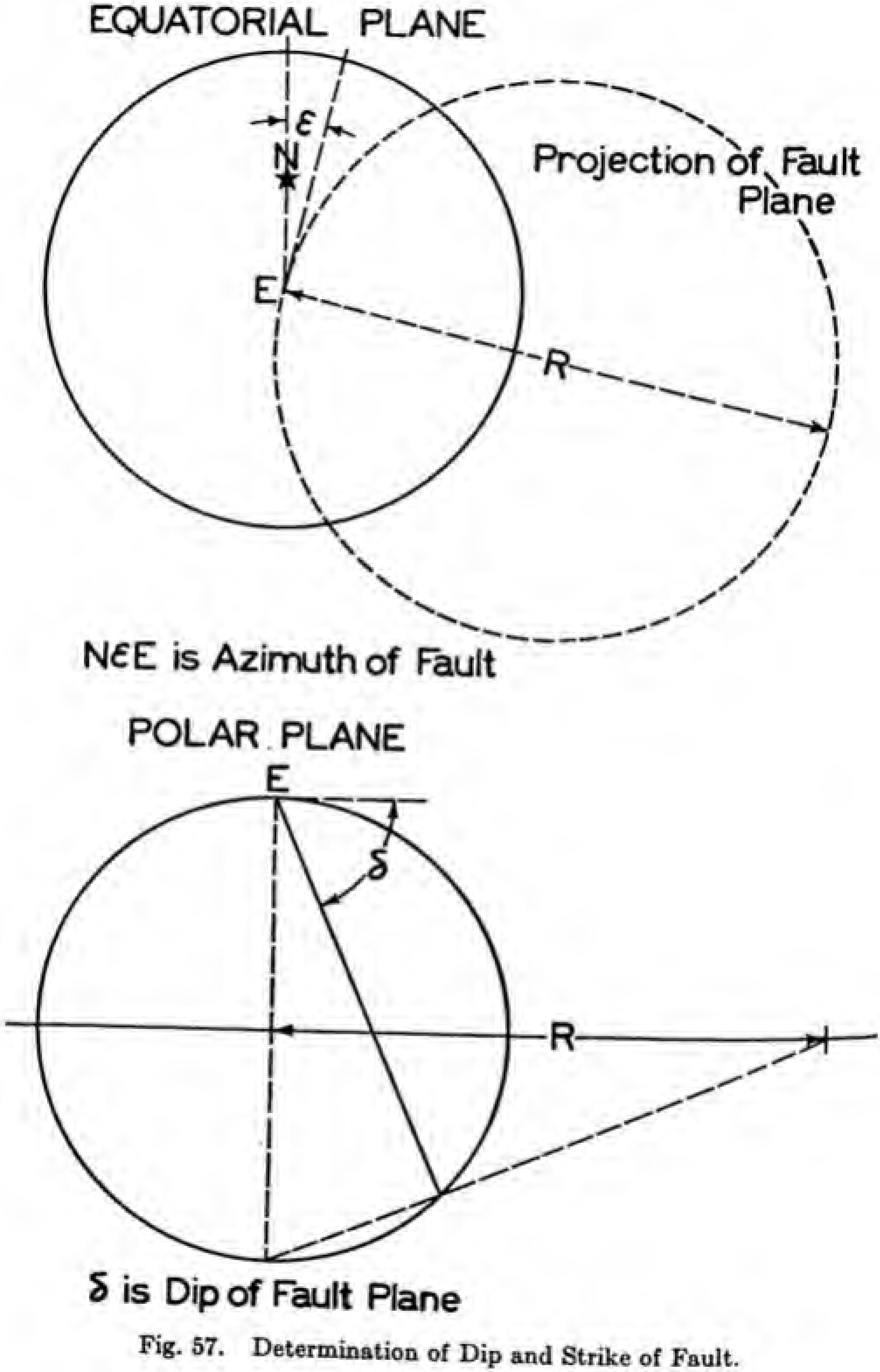

Fig. 57. Determination of Dip and Strike of Fault.

A number of Japanese seismologists have interested themselves in the problem of deducing the nature of the motion at the focus from the distribution of first motions in P and S. They have in many cases concluded sources more complicated than one built of a single couple. An excellent summary of the work has been given by Kawasumi[5].

Energy in various groups. There is little doubt that in the future comparisons of energy in the various groups of waves will lead to constructive results in seismology. It is tedious work, and careful studies will require many integrations of records. The low amplitudes of the surface waves in certain earthquakes were noted a score of years before it was realized that they indicated very deep foci for the shocks in question. A comparison of the energy in the preliminaries with that in the surface waves for various depths would no doubt be instructive.

A comparison of the energy in various groups as a function of the direction from the focus would lead to a clearer picture of what happens at the source. This comparison should be made in regard to a shock which has a good azimuthal distribution of stations about the epicenter to determine whether there are directions in which P sinks to a minimum while S rises to a maximum as might be expected from horizontal motion on a fault.

The seismograms of coastal California earthquakes written at Tinemaha and Haiwee (California) in the Owens Valley show no record of P_n, the wave which travels in the mantle just below the surface layers at these epicentral distances. The conclusion is that P_n is here blocked by a root of the southern Sierra Nevada which extends below the level of normal surface layering and consists of the lighter surface rock. Tinemaha and Haiwee must lie in the shadow of the root.

The behavior of the S group has not been studied sufficiently in detail. There appear to be subdivisions in the group which show vibrations in different planes.

Workers of the Göttingen school under Wiechert emphasized variations in the ratio of the amplitude of *PP* to that of *P* as a function of epicentral distance. At some distances *PP*, even though it has suffered loss of energy by reflection, is stronger than *P*. The weakness of *P* was attributed to changes in the rate of change of speed with depth. Rays in a layer in which speed increases rapidly are sharply curved. A cone of *P* waves which in its lower reaches is bounded above by a rapidly curving ray and below by one nearly straight would be widened and spread over a larger area of the earth's surface. *P* would then be weak where such a cone emerged.

Gutenberg and Richter have recently stressed the ratio of amplitudes of *PP* to *P* in another way. They maintain that this ratio is smaller for shocks so located that the reflection occurs under the Pacific than for those reflecting elsewhere. They conclude that this small ratio indicates a lack of continental layering under that ocean. Their reasoning is as follows: If the surface has a low-speed layer at the point of reflection, a wave approaching the surface from a given distance will be refracted toward the vertical and thus strike the free surface at a smaller angle of incidence than it would if the surface were not so layered. The smaller angle of incidence makes for more reflected energy of the same type as the incident wave. The validity of this argument depends on the length of the wave being not greater than the thickness of the layer or layers. The angle of reflection of *PP* depends only on the speed at greatest depth along a symmetrical ray and the effective speed at the surface, and not on whether the change in speed with depth is discontinuous or continuous. The effective speed "at the surface" must depend on the relation of wave length to the thickness of low-speed material near the surface. For example, the surface speed of earthquake waves is never the speed in the concrete pier on which the seismometer sits; the wave length is so vastly greater than the thickness of the pier as to render the latter neg-

ligible. These comparisons of the ratio of the amplitude of *PP* to that of *P* have assumed equal energy radiated from the source in the direction of the exodus of *PP* and of *P*. An investigation of Alexis I. Mei, S.J., as yet unpublished, throws new doubt on this assumption. He finds for certain Mexican earthquakes the epicenters of which are equidistant from Berkeley and Florissant that the ratios of the amplitudes of *PP* to *P* are quite different at the two stations. Moreover, the differences are regular.

For studies of the ratio of amplitudes of reflected to those of direct waves it is important that the recorded waves along the two paths be of the same period. Oddly, they are not always so observed. Presumably both periods are present in the direct wave, one masking the other, but the former is lost along the reflected path. The appearance of the record sometimes fails to encourage this explanation.

The ratio of horizontal to vertical components of the amplitude of Rayleigh waves is related to the gross geological foundation, according to the work of Lee[6]. For thin layers of clay or granite (thickness small compared to a wave length) this ratio may vary from 0.68 to 1.35 as layer thickness varies from 0 to 1 kilometer. For limestone or granite a thin layer of thickness 1 kilometer gives a ratio of about 0.96.

Periods

At the source of an earthquake there is an irregular breakage of rock masses, and the resulting motion is a series of jerks. We must not allow ourselves to think of the resulting waves as simple harmonic even though we are frequently forced to assume such motion in order to be able to solve the differential equations of seismograph and wave propagation. Earthquake waves are not periodic—the motion is not repeated at given intervals of time. However, in some portions of the record we do find short trains of waves which bear some semblance to sine waves. If the wave is not too irregular, we speak of its period as the time

on the seismogram between two successive crossings of the zero line going in the same direction.

The jerking motion along the fault sends out elastic waves which are no doubt complicated by internal reflections in the surface layers. There is doubtless some smoothing of irregularities in the mechanism of propagation. The period of the conspicuous waves recorded by any seismograph is a function of the free period of that seismograph. This does not mean that the seismograph, if well damped, introduces motion not present in the earthquake. It means that the seismograph magnifies those components of the complicated earth motion which have periods near or less than its own and tends to neglect those motions of period much greater than its own. A seismometer which uses galvanometric recording also neglects periods much less than its own.

Small earthquakes do not produce large long period waves, but in them short period waves (say periods of a second or less) dominate. Large earthquakes contain both long and short period motions. (*P* may have periods as large as 25 seconds or even more; *G* may have periods at least as long as 3 minutes.) Short period waves are damped in propagation more than long period waves. Larger shocks are not well recorded near the epicenter (except on the new strong motion seismographs), for the motion is too great. At the greater distances where they are well recorded, ordinary long period ($T_0 = 12$ sec. circa) seismographs record little of the short period motion which has been largely damped out. So we have learned to associate long periods with distant earthquakes and short periods with local earthquakes. The newer short period, high magnification Benioff seismographs have shown, however, that short period waves of small earth amplitude persist to great distances.

Born[14] has shown by laboratory experiment that for dry rocks the energy loss per cycle due to internal damping is independent of the frequency within the frequency range

of about 1,200 to 10,000 cycles per second and therefore is caused by internal friction rather than viscosity. This independence leads to an attenuation for amplitudes of waves in dry rock of the type $e^{-K/t}$, where K is a constant, f the frequency, and t the time. The result will be a relatively rapid damping of high frequency waves, as is normally observed. Rocks with a high moisture content exhibit a viscous type of internal damping, the damping factor K depending on the frequency of the vibration. We note that Born's experiments were for frequencies much higher than those observed in natural earthquakes.

We do not know the limits of the periods of earthquake waves. For exceedingly short waves the amplitude must be very small (to prevent rock rupture), and they would thus fail to record. No doubt waves down to molecular dimensions are possible near the source. For very long waves the magnification of our seismographs is very small, and these would thus be lost. The theoretical limit of such wave lengths is determined by the dimensions of the earth.

We see here the danger of speaking of any pendulum seismometer as an "accelerometer," for this term infers that its free period is much shorter than that of any wave it will ever record, or of calling it a "displacement-meter," which requires a free period much longer than that of any wave ever to be recorded. For a particular earthquake at a particular distance a given seismometer may act as the one or the other, but not for all earthquakes.

Dispersion (variation of speed with period) has been observed only in the surface waves. Longer waves penetrate to greater depths where the speed is greater and therefore travel with a higher speed than short waves. For example, G waves (Love waves) of period 1 minute travel with a speed of about 4.6 km/sec under the Pacific, whereas those of period 20 seconds travel with a speed of about 4.2 km/sec under the same ocean. The speeds of G waves under continents are somewhat lower, about 4.3 km/sec and 3.3 km/sec, respectively From the various theories

of the propagation of Love waves and the data of dispersion, or speed as a function of period, one can compute something as to the variation of the speed of shear waves with depth. For instance, from Love's original theory, if the speeds of S and the rigidities of the layer and the underlying material are known, the thickness of the layer may be estimated from the dispersion curve. For the speeds for upper and lower medium may be taken the speeds for the very long and the very short Love waves,

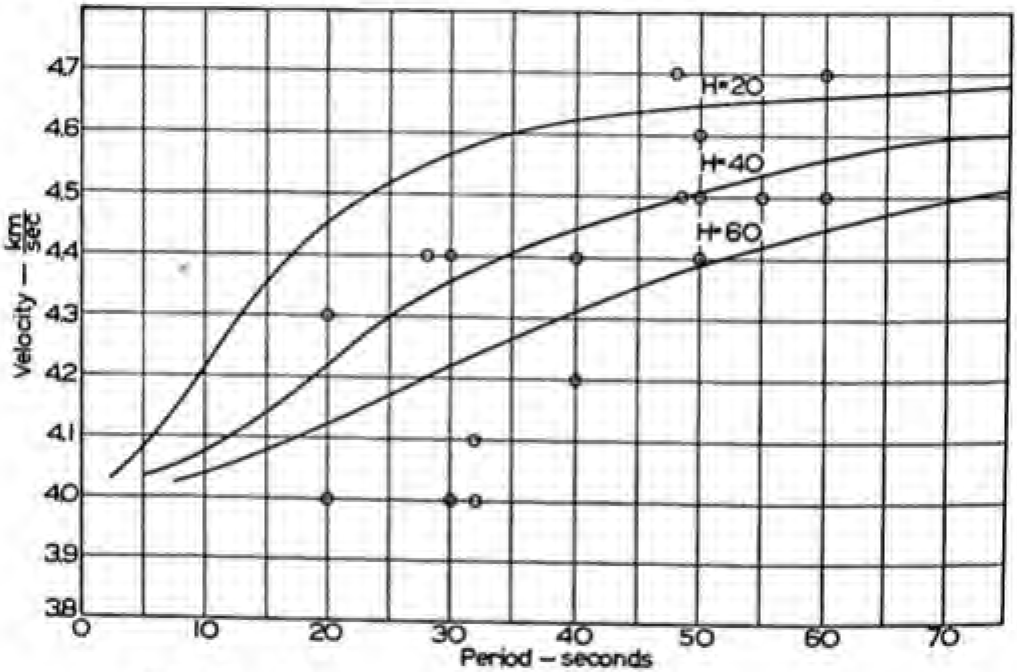

Fig. 58. Dispersion of Love Waves under the Pacific.

since these are the values approached by them. To get the rigidities the densities must be estimated. Figure 58 shows dispersion data for Love waves under the Pacific together with the theoretical curves for layers of thickness of 20, 40, and 60 kilometers[7]. These curves were obtained from equation 79. The periods and speeds plotted are those of the first crest in G and are taken as giving the wave velocity, not the group velocity. The equation applies to wave velocity. If the period and speed measured were those of a wave in the middle of a group of waves of periods not greatly different, the velocity computed would be a

group velocity. When the wave measured is the first, the question of which velocity has been observed is not so clear. The speed of the first wave varies with its period, and the phase of the first motion of G did not change with epicentral distance in the only case studied as to such behavior, the Chilean earthquake of 1922.

The dispersion of Rayleigh waves in a layered medium and in media of continuously varying speeds has been studied by Sezawa[8], who has concluded a layer (probably of basalt) 7 kilometers thick under the Pacific and two layers totaling about 40 kilometers in thickness under the Eurasian continent. For a layered earth Sezawa and Kanai[9] conclude from theory that the M wave curve has two branches; after the normal Rayleigh wave there should appear at certain epicentral distances a shorter period, lower amplitude wave in which the motion of a particle in its orbit is progressive or in the opposite sense to that in the normal Rayleigh wave. They give a sample record in which they seek to identify the later M, or M_2, as they call it.

Bullen[10], studying the dispersion of Rayleigh waves in the records of a Bering Sea earthquake recorded at Wellington, decided that a crustal layer of 18 to 26 kilometers thickness under the Pacific suitably explained the dispersion of Rayleigh waves.

Studies of the dispersion of surface waves have indicated a higher surface speed under the Pacific than under continents.[11] Whether the Atlantic basin is different from the Pacific in this way is still questioned. Gutenberg and later Gutenberg and Richter have found it more like the continents. Wilson[12] finds it like the Pacific in showing high surface speed.

There has been considerable discussion as to whether or not certain parts of the earth's crust have "free periods" of their own. There is no theoretical ground for expecting such periods. Such theory as exists points out that for a given angle of incidence below a given surface layer there

will be certain periods for which the internally reflected wave will reinforce the direct wave. Since this period depends on the angle of incidence, it can scarcely be termed a free period of the terrain. Observations by competent strong motion seismographs have not been made for long enough to indicate whether or not successive earthquakes in a given locality show predominance of a given period. Such meager data as exist indicate 0.3 second as a common predominant period in many localities. Many observations have been made in which a shaking machine is located on the ground and a seismograph placed near it. The speed of the shaker is varied, and at some speeds the seismograph writes larger amplitudes than at others. The bigger record has been taken to indicate resonance, a "free period" of the ground being the same as that of the shaker. In some cases these periods have been found to vary with distance of shaker from seismograph, indicating interference phenomena. In a recent paper Washburn and Wiley[13] have shown that the resonance frequency of a short period prospecting seismograph is very greatly affected by the way it is placed on the soil. For example, a particular instrument placed on the surface of a muddy slough gave peak response at 65 cycles, whereas when it was placed on wooden posts driven a foot into the mud (not to the base of the mud) its resonance was at over 300 cycles.

If certain periods are common to a given region in large shocks, they will in time become evident from their predominance in strong motion records of large shocks in that region. At present the best rule is to remember that the earthquake period to fear in any region is the free period of the structure in which one is interested. Fortunately earthquake motion is complex. If it were harmonic, resonance would indeed be a grave danger. Experience has shown that structures carefully designed and carefully built by engineers and builders who have earthquakes in mind do not suffer greatly in earthquakes of the intensities experienced in the United States.

Multiplicity of wave groups

Mohorovičić long ago suggested that the P wave was composed of several groups of waves of slightly different travel times, and that the one recorded depended on the sensitivity of the seismograph and the epicentral distance. He offered no explanation as to the reason for such multiplicity. In recent work Gutenberg and Richter have suggested that many of the recognized groups are multiple, and have drawn several parallel travel time curves for them. Establishment of multiplicity is difficult. If two groups of waves arrive within a few seconds of each other, and data for the arrival times of such waves are plotted for many shocks on one graph, the small errors of epicenters and times will cause the points to lie in a zone rather than on two discreet curves. Curves may be drawn through such scattered points in a number of ways. And such scattering might be caused by errors in observation of one group of waves.

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